Practice test 2

1. Areas between curves.
   - Find the area bounded between the curves \( y = t^2 \) and \( y = t \).
   - Find the area bounded between the curves \( y = x^2 \) and \( y = 1 \).
   - Find the area bounded between the curves \( y = 2t - t^2 \) and \( y = t \).
   - Find the area bounded between the curves \( y = 6 - x^2 \) and \( y = 4 - x \).
   - Find the area bounded between the curves \( y = x^2 \) and \( y = 2 - x^2 \).
   - Find the area bounded between the curves \( y = x^3 + 1 \) and \( y = x + 1 \).
   - Find the area bounded between the curves \( y = 1/2x^3 \) and \( y = 2x \).
   - Find the area bounded between the curves \( y = (1/9)(x^3 + x^2) + 3 \) and \( y = x + (1/9)x^2 + 3 \).
   - Find the area bounded between the curves \( y = 2x^2 - x^3 + 3 \) and \( y = 5 - x \).
   - Find the area bounded between the curves \( y = x^3 - 2x^2 + 4 \) and \( y = x + 2 \).

2. Volumes.
   - Find the volume of the solid of revolution formed when the region bounded between \( x = 1, x = 3, y = x^2 \) and \( y = 0 \) is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( x = 0, x = \pi, y = \sin(x) \) and \( y = 0 \) is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( x = \pi/4, x = \pi/2, y = \sin(x) \) and \( y = 1/2 \) is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( x = 2, x = 5/2, y = x^2 \) and \( y = 2 \) is revolved vertically around the line \( y = 1 \).
   - Find the volume of the solid of revolution formed when the region bounded between \( x = 1, x = 2, y = e^{x/3} \) and \( y = 1 \) is revolved vertically around the line \( y = -1 \).
   - Find the volume of the solid of revolution formed when the region bounded between \( y = x + 5 \) and \( y = x^2 + 3 \) is revolved horizontally around the y-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( y = 2x - 6 \) and \( y = (x - 3)^2 \) is revolved horizontally around the line \( x = 1 \).
   - Find the volume of the solid of revolution formed when the region bounded between \( y = 6 - 2x \) and \( y = (x - 2)^2 + 2 \) is revolved vertically around the line \( y = -1 \).
   - Find the volume of the solid of revolution formed when the region bounded between \( y = x \) and \( y = x^2 \) is revolved horizontally around the line \( x = -2 \).

3. Volumes by cylindrical shells.
   - Find the volume of the solid of revolution formed when the region below the curve \( y = x^2 \), above the x-axis, and between \( x = 0 \) and \( x = 1 \) is revolved around the y-axis.
   - Find the volume of the solid of revolution formed when the region below the curve \( y = \cos(x) \), above the x-axis, and between \( x = 0 \) and \( x = \pi/2 \) is revolved around the y-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( y = 3x + 2 \) and \( y = x^2 + 2 \) is revolved horizontally around the y-axis.
   - Find the volume of the solid of revolution formed when the region bounded between \( y = x + 2 \) and \( y = x^2 \) is revolved horizontally around the line \( x = -2 \).
   - Find the volume of the solid of revolution formed when the region bounded between \( y = x - 1 \) and \( y = (x - 1)^2 \) is revolved vertically around the line \( y = -1 \).

4. Work
1. A spring whose natural length is 3 ft exerts a force of 1.2 lbs when stretched 1.1 ft beyond its natural length. How much work is required to stretch the spring from its natural length to a length of 3.8 ft?

2. A spring whose natural length is 1 m exerts a force of 1.5 N when stretched 2 m beyond its natural length. How much work is required to compress the spring from its natural length to a length of 1.6 m?

3. Suppose that a cylindrical tank has height 10, the radius of the base is 7, and it is half filled with water. Find the amount of work necessary to move all of the water out of the top of the tank.

4. Suppose that a tank conical in shape with the cone point on the bottom has height 10, the radius of the base is 7, and it is half filled with water. Find the amount of work necessary to move all of the water out of the top of the tank.

5. Find the amount of work to move a tank up to the top of a building which is 250 feet high. The tank is 5 foot high and weighs 200 pounds. A cable is attached to the top of the tank and a winch on the top of the building; it weighs 3 pounds per foot.

6. One of the bouldering problems at our climbing gym is called “crazy dyno” – to solve this problem you need to throw yourself into air, and reach with your left hand for a hold located approximately half a meter to the left and a meter and a half above the jug where your left hand rests when you start the problem. Assuming that your weight is 80 kg, and that you have one meter of space to take the swing before you jump, how much power (recall that power = work/time) do you need to send the crazy dyno?

5. **Average value of a function.**

   1. Find the average value of $f(x) = x$ on the interval [0,1].
   2. Find the average value of $f(x) = x^2 + 1$ on the interval [1,3].
   3. Find the average value of $f(x) = x^2 - x$ on the interval [0,1].
   4. Find the average value of $f(x) = \sin x$ on the interval [0, π].
   5. Find the average value of $f(x) = 3$ on the interval [5,18].