8 Rational function field of an affine algebraic variety. Birational equivalence of affine algebraic sets.

1. Let V be the affine variety $\mathcal{Z}(x^2+y^2-1)\subseteq k^2$ and consider the rational function $\varphi=\frac{y+1}{x}$ on V. Determine at which points φ is regular (note: beware of $\mathrm{char}(k)$).

2. Is the function $\frac{x}{y}$ regular at the point $(0,0)$ of the variety $\mathcal{Z}(x)\subseteq k^2$?

3. At which points of the curve C defined by $y^2=x^2+x^3$ is the rational function $t=\frac{y}{x}$ regular? Prove that $\frac{y}{x}\notin k[C]$.

4. Prove that $\mathcal{Z}(x^2+y^2-1)\subseteq k^2$ is a rational curve.

5. Prove that any conic section in k^2 is a rational curve.

Homework: Problems 1, 2 and 3.