7 Morphisms of affine algebraic sets. Category of affine algebraic sets. Isomorphisms of affine algebraic sets.

1. Prove that the hyperbola defined by $xy = 1$ and the line k^1 are not isomorphic as affine algebraic sets.

2. An isomorphism $f: V \to V$ of an affine algebraic set V is called an **automorphism**. Prove that all automorphisms of the line k^1 are of the form $f(x) = ax + b$ with $a \neq 0$.

3. Prove that the map $f(x,\ y) = (\alpha x,\ \beta y + P(x))$, where $\alpha,\ \beta \in k^\times$, $P(x) \in k[x]$, is an automorphism of k^2. Prove that all maps of that form a group. We shall call this group the group of **triangular** automorphisms.

4. Let V be an affine algebraic set consisting of two points. Prove that $k[V] \cong k \times k$.

5. Let $f: V \to W$ be a morphism of affine algebraic sets. The subset $\Gamma_f \subseteq V \times W$ consisting of all points of the form $(v, f(v))$ is called the **graph** of f. Prove that Γ_f is isomorphic to V.

6. Let $\operatorname{char} k = p$. Prove that the morphism $\psi: k^n \to k^n$ given by $\psi(x_1, ..., x_n) = (x_1^p, ..., x_n^p)$ is a bijection, but not an automorphism. We shall call it the **Frobenius automorphism.**

Homework: Problems 4, 5 and 6.