# Foundations of algebraic geometry

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## Course material

1. Noetherian rings. Hilbert basis theorem.
2. Primary decomposition. Radical of an ideal.
3. Affine algebraic sets and their ideals. Zarisski topology.
4. Affine algebraic varietes. Hilbert Nullstellensatz.
5. Applications of Nullstellensatz. Maximal ideals in polynomial rings. Radical ideals.
6. Coordinate ring of an affine algebraic set.
7. Morphisms of affine algebraic sets. Category of affine algebraic sets. Isomorphisms of affine algebraic sets.
8. Rational functions field of an affine algebraic variety.
9. Rational maps of affine algebraic sets. Birational equivalence of affine algebraic sets.
10. Projective space. Projective algebraic sets.
11. Morphisms of projective algebraic sets. Rational maps of projective algebraic sets.
12. Spectrum of a ring. Zarisski topology on a spectrum, irreducibility.
13. Presheaves, structure of presheaves. Sheaves. Stalks of sheaves.
14. Schemes. Glueing of schemes.
15. Varietes.

**Literature:**

1. K. Szymiczek. *Algebra. Wykłady dla studiów doktoranckich.* `http://www.math.us.edu.pl/zatl/szymiczek/referaty/`
2. M. F. Atiyah, I. G. Macdonald. *Introduction to commutative algebra.* Reading, Mass., 1969.
3. I. R. Shafarevich, *Basic algebraic grometry vol. 1 & 2*. 3rd Edition. Springer, 2013.
4. R. Hartshorne. *Algebraic geometry.* Springer, 1977.

**Grade:**

- 2 tests, each worth 30%
- homework assignments, worth 30% total
- class participation, worth 10%

**Office hours:**

- Wednesdays, 13:00 – 13:45

If you want to talk to your instructor, let him know before or after the class, send him an email, or call his office.

Other that the abovelisted days/times are also possible via Zoom/Teams/etc.

# 1 Noetherian rings.

## 1.1 Noetherian rings.

**Theorem 1.1.** *Let $R$ be a ring. The following conditions are quivalent:*

***(FG).*** [1.1] *Every ideal of the ring $R$ is finitely generated.*

***(ACC).*** [1.2] *Every ascending chain of ideals in $R$ is finite.*

***(MAX).*** *Every nonempty family of ideals of the ring $R$ has a maximal element.*

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1.1. finitely generated

1.2. ascending chain condition

**Definition 1.2.** *Let $R$ be a ring. If one (and hence every) condition of Theorem 1.1 is satisfied, then $R$ is called a* ***noetherian ring.***

**Lemma 1.3.** *Let $R$ and $S$ be rings and let $R$ be noetherian. Let $\varphi\colon R \to S$ be an epimorphism. Then $S$ is noetherian.*

**Corollary 1.4.** *Let $R$ be noetherian, let $I \lhd R$. Then $R/I$ is noetherian.*

## 1.2 Hilbert basis theorem.

**Theorem 1.5. (Hilbert basis theorem)** *Let* $R$ *be noetherian. Then* $R[x]$ *is noetherian.*

**Corollary 1.6.** *Let $R$ be noetherian. Then $R[x_1, ..., x_n]$ is noetherian.*