

6 Coordinate ring of an affine algebraic set.

1. Is the ring $\mathbb{Z}$ isomorphic to a coordinate ring of an affine algebraic set?
2. Let $V = \mathcal{Z}(xy - 1) \subseteq \mathbb{C}^2$. Show that $\mathbb{C}[V] \cong \mathbb{C}[x, \frac{1}{x}]$.
3. Show that coordinate rings of a circle and a hyperbola considered as affine algebraic sets in $\mathbb{C}^2$ are isomorphic.
4. Let $V = \mathcal{Z}(x^2 + y^2 - z^2) \subseteq \mathbb{C}^3$. Find $\mathbb{C}[V]$.
5. Let $V = \mathcal{Z}(x^2 + y^2 - z^2) \subseteq \mathbb{C}^3$, let $f = x^3 + 2xy^2 - 2xz^2 + x$ and $g = x - x^3$. Show that $f_V = g_V$.
6. Let $V = \mathcal{Z}(y^2 - x^3)$. Show that an element of $k[V]$ can be written uniquely in the form $p(x) + q(x)y$ with $p, q \in k[x]$.
7. Let $V = \{(t, t^2, t^3) | t \in k\}$. Show that V is an affine algebraic set and prove that $k[V] \cong k[x]$.

Homework: Problems 5, 6 and 7.