1. Problem set 1: Modules, submodules, submodules generated by sets.

(1) Let R be a ring, M a left R-module, $\mathcal{R} = \{N_i : i \in I\}$ a family of submodules of M. Show that:
   (a) $\bigcap_{i \in I} N_i$ is a submodule of M,
   (b) $\bigcup_{i \in I} N_i$ is a submodue of M if $\mathcal{R}$ is a chain.
   If $\bigcup_{i \in I} N_i$ is a submodue of M, is $\mathcal{R}$ necessarily a chain?

(2) Let R be a ring, M a left R-module, let $S \subset M$. Show that:
$$\langle S \rangle = \{a_1 s_1 + \ldots + a_n s_n + b_1 t_1 + \ldots + b_m t_m : a_i \in R, s_i \in S, b_j \in \mathbb{Z}, t_j \in S\}.$$

(3) Let R be a ring, M a left R-module, let $N_1, N_2 < M$. Show that
$$\langle N_1 \cup N_2 \rangle = \{n_1 + n_2 : n_1 \in N_1, n_2 \in N_2\}.$$

(4) Give an example of a finitely generated module, which is not a finitely generated Abelian group.

(5) Let R and S be rings, let $\phi : R \to S$ be a ring homomorphism. Show that every S-module M is also an R-module by defining rm, $r\ inR$, $m \in M$ as $\phi(r)m$.

(6) Let R be a ring, $I \lhd R$ a two-sided ideal, M a left R-module. Show that M/IM is a R/I-module with $(r + I)(a + IA) = ra + IA$.

Homework: Problems 3,4,6 are to be handed in during the next class.