**Problem set 21: triple integrals.**

(1) Let $C$ be the cube $[-1,1]^3$. Compute $\iiint_C x^2y^2z^2\,dV$.

(2) Let $T$ be the tetrahedron spanned be the vectors $e_1$, $e_2$, and $e_3$. Calculate its volume by a triple integral.

(3) Find the cylindrical coordinates of the point with rectangular coordinates $(-4,4,3)$.

(4) Describe the surface whose equation in cylindrical coordinates is $z=2r$.

(5) Use cylindrical coordinates to compute the volume of a cylinder with the $z$-axis as its axis and bounded by the planes $z=0$ and $z=1$.

(6) Evaluate $\int_0^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^3(x^2+y^2)\,dz\,dy\,dx$.

(7) Evaluate the following integral by changing to cylindrical coordinates:

$$\int_{-5}^5\int_0^{\sqrt{25-x^2}}\int_0^{25-x^2-y^2}(x^2+y^2)\,dz\,dy\,dx.$$

(8) Compute the volume of a ball with radius 1 with help of spherical coordinates.

(9) Compute the volume of the spherical wedge given by

$$\Omega=\{(\rho,\theta,\phi):a\le\rho\le b,\alpha\le\theta\le\beta,c\le\phi\le d\}.$$

(10) Find the volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=2z$.

(11) Evaluate $\iiint_B\sin((x^2+y^2+z^2)^{3/2})\,dV$, where $B$ is the unit ball.

(12) Write the following equations in spherical coordinates:
(a) $z^4=x^2+y^2$;
(b) $x^2-3x-4y=z^2$.

(13) Evaluate $\iiint_B(x^2+y^2+z^2)^6\,dV$, where $B$ is the ball centered at the origin with radius 2.

(14) Calculate $\iiint_A((x-1)^2+y^2+z^2)\,dV$, where $A$ is the annulus centered at $(1,0,0)$ with radii 1 and 2.

(15) Write five other integrals that are equal to $\int_0^1\int_x^1\int_0^x f(x,y,z)\,dz\,dy\,dx$.

(16) Is the following equality true

$$\int_0^3\int_0^2\int_0^y\sqrt{y+x^2}\,dx\,dy\,dz=\int_0^3\int_0^y\int_0^2\sqrt{y+x^2}\,dy\,dx\,dz?$$

Justify your answer.

(17) Is the following identity true

$$\int_0^5\int_0^\pi\int_{-\pi}^\pi\cosh(z^3)(z+z^3+\sin(z))e^{y^2}\tan(x)\,dz\,dy\,dx=0?$$

(18) Show that

$$\lim_{t\to1-}\int_0^t\int_0^t\int_0^t\frac{1}{1-xyz}\,dx\,dy\,dz=\sum_{n=1}^\infty\frac{1}{n^3}.$$

(19) Show that Laplace's equation

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=0$$

written in cylindrical coordinates becomes

$$\frac{\partial^2u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2u}{\partial\theta^2}+\frac{\partial^2u}{\partial z^2}=0.$$

(20) Assume that $f\colon \mathbb{R} \to \mathbb{R}$ is continuous. Show that $\int_0^{x_2} \int_0^{x_1} \int_0^{x_0} f(t)\,dt\,dx_0\,dx_1 = \frac{1}{2} \int_0^{x_2} (x_2 - t)^2 f(t)\,dt$.