

Problem set 17: definite integrals and applications.

- (1) Use integration to calculate the area of a circle.
- (2) Determine $b > 0$ such that the area below $1/x$ and contained in the set $\{(x, y) : 0 \leq y, 1 \leq x \leq b\}$ equals 2.
- (3) Find a curve $y = y(x)$ such that its length on $(0, 1)$ is infinite.
- (4) Find the area of the region enclosed by the parabola $y = 3 - x^2$ and $y = -x$.
- (5) Find the area of the region that is bounded from above by $y = \sqrt{x}$ and below by the x -axis and the line $y = x - 6$.
- (6) Assume we have a square-based pyramid with side length $2s$ and height h . Deduce the formula for the volume.
- (7) The region between the graph of $f(x) = 2 + x \sin x$ and the x -axis over the interval $[-2, 2]$ is revolved about the x -axis to generate a solid. Find the volume of the solid.
- (8) Find the area between the x -axis and the graph of $f(x) = |\cos(x)|$ over $[0, \pi]$.
- (9) Compute the area between the x -axis and e^{-x} given on the positive real numbers.
- (10) Find a Riemann integrable function f not identically to 0 such that

$$\int_{n+1}^{n+2} f(x) dx = \frac{1}{2} \int_n^{n+1} f(x) dx$$

for all $n \in \mathbb{N}$.

- (11) The region in the first quadrant enclosed by the y -axis, the line through $x = 2\pi$, and the graphs of $y = \sin(x)$ and $y = \frac{1}{2} \sin(x)$ is revolved around the x -axis. What is the volume of the generated solid?
- (12) Let us look at $y = 1 - ax^2$ defined in the first quadrant for some $0 < a$. We have the region bounded by y from above and the x -axis from below. Determine a such that the volume of the solid generated by rotating around the x -axis is the same as rotating around the y -axis.
- (13) Let us rotate the disk $(x - R - r)^2 + y^2 \leq r^2$ around the y -axis. What is the volume of the generated torus?
- (14) Find the length of the curve $y = 5x - 4|x|$ from $x = -2$ to $x = 2$.
- (15) Find the length of the curve $y = x^2$ from $x = -2$ to $x = 2$.
- (16) Assume a car is driving with speed $\sin(\pi t)$ for $t \in [0, 1]$. What is its average speed?
- (17) Suppose a bird is flying with speed $\cos(\pi t)$ for $t \in [0, 2]$. What is the average speed?
- (18) The exponential distribution has probability density function

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ ce^{-cx}, & \text{otherwise,} \end{cases}$$

where c is a positive constant. We need $\int_{-\infty}^{\infty} f(x) dx = 1$. What is c ?

- (19) Show that $\int_{-\infty}^{\infty} e^{-x^2/2} dx$ is a finite number.
- (20) The mean of a random variable X with probability density function f is

$$E(X) = \int_{-\infty}^{\infty} xf(x) dx$$

provided the integral converges.

Let

$$f(x) = \begin{cases} \frac{1}{x^2}, & \text{if } x \geq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Show that f is a probability density function and that the distribution has no mean.