

Problem set 16: indefinite integrals II.

- (1) Compute $\int \frac{\sqrt{x}-\sqrt[4]{x}}{x^2} dx$.
- (2) Suppose that $a \neq 0$. What is $\int \frac{x}{(x^2+a^4)^n} dx$, where $n \in \mathbb{N}$?
- (3) Compute $\int \frac{x}{\sqrt{1-x^4}} dx$.
- (4) What is the primitive of $e^x/2e^x + 2$?
- (5) Determine the primitive of $\cos^4(x) \sin(x)$.
- (6) Calculate $\int \frac{1}{e^x+e^{-x}} dx$.
- (7) What is $\int x^3 e^{x^2} dx$?
- (8) Compute $\int \frac{1}{2x^2+9x-5} dx$.
- (9) Compute $\int \frac{1}{4x^2-24x+54} dx$.
- (10) Determine $\int \frac{4x^3+x^2+2x-1}{x^4-1} dx$.
- (11) Compute $\int \frac{1}{x^3-x^2-2x-12} dx$.
- (12) Find $\int \frac{1}{\sqrt[4]{4-5x}} dx$.
- (13) What is the primitive of $\frac{1}{\sin(x)}$?
- (14) Compute the primitive of $\frac{1}{\sin^2(x) \cos^2(x)}$.
- (15) Show that

$$\sin(x) = \frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)},$$

$$\cos(x) = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)},$$

$$\tan(x) = \frac{2 \tan\left(\frac{x}{2}\right)}{1 - \tan^2\left(\frac{x}{2}\right)}.$$

Then determine the primitive of $\frac{1}{3+\cos(x)}$ by using the substitution $u = \tan\left(\frac{x}{2}\right)$.

- (16) Compute $\int \left(\frac{2+\sin(x)}{(1+\cos(x)) \sin(x)} - \sin(x) \right) dx$.
- (17) Show that

$$\sin^2(x) = \frac{\tan^2(x)}{1 + \tan^2(x)},$$

$$\sin(x) \cos(x) = \frac{\tan(x)}{1 + \tan^2(x)},$$

$$\cos^2(x) = \frac{1}{1 + \tan^2(x)}.$$

Then use the substitution $t = \tan(x)$ to calculate

$$\int \frac{1}{2 + 4 \cos^2(x)} dx.$$

- (18) Compute $\int \tan(x) dx$.
- (19) Compute $\int \frac{x}{1-\sin^2 x} dx$.
- (20) Compute $\int x \tan^2(x) dx$.