## Problem set 13: derivatives.

- (1) Compute the derivative of the function  $x \mapsto 8x^8 + 3x^2 1$ .
- (2) Give an example of a function that is not constant and whose derivative vanishes everywhere in the domain.
- (3) Compute the derivative of the function  $x^2 + \sqrt{x}$  on  $(0, \infty)$ .
- (4) Is |x| defined on  $\mathbb{R}$  differentiable at 0?
- (5) Let  $f: \mathbb{R} \to \mathbb{R}$  be a function. We say that its *symmetric derivative* exists at some point x if the following limit exists:

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

Show that if the derivative exists at a point x, then so does the symmetric derivative, and the two agree at this point.

- (6) Compute the derivative of  $x^4\sqrt{x}$ .
- (7) Let  $f(x) = x^3$ . Determine the sets where f'(x) = 0, f'(x) > 0 and f'(x) < 0.
- (8) Calculate the derivative of

$$\frac{x^3 - 7x\sqrt[5]{x^2}}{2\sqrt{x}}$$

defined on  $(0, \infty)$ .

(9) What is the derivative of

$$\sqrt[3]{x^4\sqrt{x}}$$

on  $(0,\infty)$ ?

- (10) Compute the derivative of  $x^5 \cos(x)$ .
- (11) Compute the derivative of the function

$$x \mapsto \frac{1-x^2}{7x^2+9}.$$

(12) What is the derivative of

$$y = (7x^5 + 6x^3 - x^2 - 1)^3?$$

(13) What is the derivative of  $\sin^2 x + \cos^2 x$ ?

- (14) Compute the derivative of  $\sin(4x^2)$ .
- (15) Calculate the derivative of  $\sin^5 x$ .
- (16) Determine the derivative of  $e^{-2x}$ .
- (17) Compute the derivative of  $\tan^4(3x)$  where the function is defined.
- (18) What is the derivative of  $x^x$  where it exists?