Problem set 4: systems of linear equations and Gauss elimination.

(1) Solve the following systems of linear equations over $\mathbb{R}$:

(a) $\begin{cases} 2x-3y+5z+7t=1 \\ 4x-6y+2z+3t=2 \\ 2x-3y-11z-15t=1 \end{cases}$; (b) $\begin{cases} 2x+5y-8z=8 \\ 4x+3y-9z=9 \\ 2x+3y-5z=7 \\ x+8y-7z=12 \end{cases}$;

(c) $\begin{cases} 3x+4y+z+2t=3 \\ 6x+8y+2z+5t=7 \\ 9x+12y+3z+10t=13 \end{cases}$; (d) $\begin{cases} 3x-5y+2z+4t=2 \\ 7x-4y+z+3t=5 \\ 5x+7y-4z-6t=3 \end{cases}$;

(e) $\begin{cases} 3x-2y+5z+4t=2 \\ 6x-4y+4z+3t=3 \\ 9x-6y+3z+2t=4 \end{cases}$; (f) $\begin{cases} 8x+6y+5z+2t=21 \\ 3x+3y+2z+t=10 \\ 4x+2y+3z+t=8 \\ 3x+5y+z+t=15 \\ 7x+4y+5z+2t=18 \end{cases}$;

(g) $\begin{cases} x+y+3z-2t+3w=1 \\ 2x+2y+4z-t+3w=2 \\ 3x+3y+5z-2t+3w=1 \\ 2x+2y+8z-3t+9w=2 \end{cases}$; (h) $\begin{cases} 2x-y+z+2t+3w=2 \\ 6x-3y+2z+4t+5w=3 \\ 6x-3y+2z+8t+13w=9 \\ 4x-2y+z+t+2w=1 \end{cases}$;

(i) $\begin{cases} 6x+4y+5z+2t+3w=1 \\ 3x+2y+4z+t+2w=3 \\ 3x+2y-2z+t=-7 \\ 9x+6y+z+3t+2w=2 \end{cases}$.

(2) Solve the following systems of equations over $\mathbb{Q}$ and $\mathbb{Z}_p$:

(a) $\begin{cases} 2x+7y+3z+t=6 \\ 3x+5y+2z+2t=4 \\ 9x+4y+z+7t=2 \end{cases}$, $p=11$; (b) $\begin{cases} 9x-3y+5z+6t=4 \\ 9x-3y+5z+6t=4 \\ 3x-y+3z+14t=-8 \end{cases}$, $p=13$;

(c) $\begin{cases} 6x+3y+2z+3t+4w=5 \\ 4x+2y+z+2t+w=4 \\ 4x+2y+3z+2t+w=0 \\ 2x+y+7z+3t+2w=1 \end{cases}$, $p=11$; (d) $\begin{cases} 2x-y+3z-7t=5 \\ 6x-3y+z-4t=7 \\ 4x-2y+14z-31t=18 \end{cases}$, $p=37$;

(e) $\begin{cases} x+2y+3z-2t+w=4 \\ 3x+6y+5z-4t+3w=5 \\ x+2y+7z-4t+w=11 \\ 2x+4y+2z-3t+3w=6 \end{cases}$, $p=13$; (f) $\begin{cases} 3x+2y+2z+2t=2 \\ 2x+3y+2z+5t=3 \\ 9x+y+4z-5t=1 \\ 2x+2y+3z+4t=5 \\ 7x+y+6z-t=7 \end{cases}$, $p=7$;

(g) $\begin{cases} 2x+3y+z+2t=4 \\ 4x+3y+z+t=5 \\ 5x+11y+3z+2t=2 \\ 2x+5y+z+t=1 \\ x-7y-z+2t=7 \end{cases}$, $p=17$.

(3) Solve the following systems of equations over $\mathbb{Z}_5$, $\mathbb{Z}_7$, and $\mathbb{Z}_{11}$:

(a) $\begin{cases} x+4y+3z=2 \\ 3x+2y+4z=3 \\ 4x+y+z=0 \end{cases}$, (b) $\begin{cases} 2x+3y+z=1 \\ x+4y+3z=3 \\ 4x+3z=2 \end{cases}$.

(4) Show that the system of equations $\begin{cases} x+y+z=1 \\ 2x+y-z=2 \\ x-y+3z=0 \end{cases}$ has no solutions over $\mathbb{Z}_p$ if and only if $p=2$.

(5) Solve the following system of equations over $\mathbb{C}$:

$$\begin{cases} 6ix+(-3+6i)y+(4+2i)z+(1+2i)t=0 \\ (5+5i)x+(3+5i)y+(7-3i)z+(4+2i)t=0 \\ (-3+3i)x+(-6+3i)y+(-1+3i)z-t=0 \\ (1+11i)x+(1+12i)y+(11+7i)z+7it=0 \end{cases}$$

assuming that:
(a) $x=0$, (b) $y=0$, (c) $z=0$, (d) $t=0$, (e) $x+y=0$.

(6) Solve the following systems of equations over $\mathbb{C}$:

(a) $\begin{cases} (1+i)x+2iy-z=3+2i \\ (3+i)x+(1-i)y+4z=6+i \\ 5x+y-iz=2 \end{cases}$, (b) $\begin{cases} (1+i)x+2y-iz=2-3i \\ 3x+iy+(2-i)z=6+4i \\ (4+i)x+y+3z=6+6i \end{cases}$.

(7) For which values of the parameter $\lambda \in \mathbb{Z}_7$ the system of equations $\begin{cases} x+2y+6z+6t=1 \\ x+y+z+3t=2 \\ 3x+5y+6z+t=\lambda \end{cases}$ over $\mathbb{Z}_7$ has a solution?

(8) Depending on the parameter $\lambda \in \mathbb{Q}$ solve the following systems of equations:

(a) $\begin{cases} 8x+6y+3z+2t=5 \\ -12x-3y-3z+3t=-6 \\ 4x+5y+z+4t=3 \\ \lambda x+4y+z+4t=2 \end{cases}$, (b) $\begin{cases} 2x-y+3z+4t=5 \\ 4x-2y+5z+6t=7 \\ 6x-3y+7z+8t=9 \\ \lambda x-4y+9z+10t=11 \end{cases}$, (c) $\begin{cases} \lambda x+y+z+t=1 \\ x+\lambda y+z+t=1 \\ x+y+\lambda z+t=1 \\ x+y+z+\lambda t=1 \end{cases}$.