

Name:

Perm number:

**Midterm – take-home part**

Time: 75 minutes

1. **(25 points)** Bernoulli equation. Consider the equation

$$\frac{dy}{dx} + p(x)y + q(x)y^n = 0$$

- Show that by substituting  $z = y^{1-n}$  one obtains a linear equation.
- Find all solutions to the equation

$$y' + y + x\sqrt{y} = 0$$

2. **(25 points)** Riccati equation. Consider the equation

$$\frac{dy}{dx} = p(x)y^2 + q(x)y + r(x)$$

- Show that if  $y_1(x)$  is one of the solutions of the Riccati equation, then by substituting  $y = y_1(x) + \frac{1}{u}$  ( $u$  is a new variable here) one obtains a linear equation.
- Find all solutions to the equation

$$\frac{dy}{dx} = y^2 - (4x + 1)y + 4x^2 + 2x + 2,$$

knowing that  $y_1(x) = 2x$  is one of the solutions.

- Find all solutions to the equation

$$\frac{dy}{dx} + y^2 - 1 = x^2$$

(you'll have to start by guessing one of the solutions!).

3. **(50 points)** Clairaut equation. Consider the equation

$$y = xy' + f(y').$$

- Develop an algorithm of solving Clairaut equations. Start with taking derivatives of both sides of the above equation and carefully considering two cases.
- Find at least one solution to the equation

$$y = xy' + (y')^4.$$

- Find all solutions to the equation

$$y = xy' + (y')^4.$$

Be careful! Constants that you'll get in the process of integration are not independent of each other (why?)!