Name: **Perm number:**

Final exam – take-home part

1. **(10 points)** Solve the following three systems of equations.

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} , \quad \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ,$$

$$\text{and} \quad \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

2. **(10 points)** Next find a matrix that solves the following equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & x_3 \\ z_1 & z_2 & z_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. **(10 points)** Row-reduce the following matrix to find a reduced row-echelon form:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

4. **(15 points)** Take your solution matrix to problem 2. and calculate the product in reverse order.

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & x_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} = ?$$

5. **(20 points)** Use your observation in 4 to solve the following system without row operations:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$$

6. **(35 points)** In the above you found how to find the inverse of a square matrix. Find the inverses of the following by using the ideas in problem 3. Check your answers.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 3 & -1 \\ -7 & 2 \end{pmatrix},$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix},$$

$$\begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ 1 & 1 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 \\ 0 & -1 & 2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2 & -1 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{pmatrix}$$