Practice Test 1

Decide if the following statements are true or false:

(1) $(p \wedge r) \wedge (\neg q \to s) \wedge ((r \wedge s) \to \neg t) \wedge (p \wedge \neg q) \to (\neg t \wedge s)$
(2) $(\neg q \wedge s) \wedge (p \wedge \neg t) \wedge ((\neg t \wedge s) \to u) \wedge ((\neg q \wedge p) \to z) \to u \wedge z$
(3) $(p \wedge q) \wedge r \leftrightarrow p \wedge (q \wedge r)$
(4) $(p \to q) \wedge (p \to (q \to r)) \to (p \to r)$
(5) $(p \to (q \to r)) \to (q \to (p \to r))$
(6) $((p \to q) \to r) \to (p \to (q \to r))$
(7) $(p \to q) \wedge (r \to s) \to ((p \wedge r) \to (q \wedge s))$
(8) $(p \to q) \wedge (r \to s) \to ((p \wedge q) \to (r \wedge s))$
(9) $(p \vee q) \vee r \leftrightarrow p \vee (q \vee r)$
(10) $((p \vee q) \to r) \leftrightarrow ((p \to r) \wedge (q \to r))$
(11) $(p \to q) \wedge (r \to s) \to ((p \vee r) \to (q \vee s))$
(12) $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
(13) $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$
(14) $p \wedge (q \vee r) \leftrightarrow (p \wedge r) \vee (q \wedge r)$
(15) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
(16) $(p \to q) \leftrightarrow (\neg p \to \neg q)$
(17) $(p \to q) \wedge \neg q \to \neg p$
(18) $((p \wedge q) \to r) \leftrightarrow ((p \wedge \neg r) \to \neg q)$

Set theory:

(1) Out of a total of 30 students, 19 are doing mathematics, 17 are doing music, and 10 are doing both. How many are doing neither?
(2) How many integers are there between 1000 and 999 that contain the digits 0, 8, and 9 at least once each?
(3) 73% of British people like cheese, 76% like apples, and 10% like neither. What percentage like both cheese and apples?
(4) In a class of 30 students, 16 cheer for Manchester United, 17 cheer for Liverpool, and 14 for Chelsea; also 8 cheer for both Man United and Liverpool (wonder how that's possible...), 7 for Man and Chelsea, and 9 for Liverpool and Chelsea. How many cheer for all three teams?
(5) On a plane there are 9 boys, 5 American children, 9 men, 7 non-American boys, 14 Americans, 6 American males, and 7 non-American females. How many people are there on the plane altogether?
(6) Find the number of integers between 1 and 5000 that are divisible by neither 3 nor 4.
(7) Find the number of integers between 1 and 5000 that are divisible by neither 3 nor 4 nor 5.
(8) Find the number of integers between 1 and 5000 that are divisible by one or more of the numbers 4, 5 and 6.

Mathematical induction:

(1) $\sum_{k=1}^{2n}(-1)^{k+1}k = -n$
(2) $\sum_{k=1}^{2n}(-1)^{k+1}k^2 = -n(2n+1)$
(3) $\sum_{k=1}^{2n}(-1)^{k+1}k^3 = -n^2(4n+3)$
(4) $\sum_{k=1}^{2n}(-1)^{k+1}k^4 = -n(2n+1)(4n^2+2n-1)$
(5) $\sum_{k=1}^{2n}(-1)^{k+1}k^5 = -n^2(16n^3+20n^2-5)$
(6) $\sqrt[n]{n} \le 1+\sqrt{2/n}$, $n \ge 1$
(7) $2 \le (1+1/n)^n \le n+1$, $n \ge 1$
(8) $(1+1/n)^n \le 3$, $n \ge 1$
(9) $\frac{n+1}{n} \le (\frac{n^2}{n^2-1})^n \le \frac{n}{n-1}$, $n \ge 2$
(10) $(n/3)^n \le n! \le (n/2)^n$, $n \ge 0$

Greatest common divisor:

(1) If $a = 17$ and $b = 29$, find $d = gcd(a,b)$ and the integers s, t such that $d = sa + tb$.
(2) If $a = 713$ and $b = 552$, find $d = gcd(a,b)$ and the integers s, t such that $d = sa + tb$.
(3) If $a = 299$ and $b = 345$, find $d = gcd(a,b)$ and the integers s, t such that $d = sa + tb$.

Complex numbers:
Find the trigonometric form of the following complex numbers:

(1) i^n
(2) $(1+i)^n$
(3) $(1+i\sqrt{3})^n$
(4) $(\sqrt{6}+\sqrt{2}+i(\sqrt{6}-\sqrt{2}))^n$
(5) $(\frac{1+i}{1+i\sqrt{3}})^n$
(6) $(\frac{2-i\sqrt{6}}{1-i})^n$

(7) $[\frac{(1+i)(1+i\sqrt{3})}{\sqrt{5}+1+i\sqrt{10-2\sqrt{5}}}]^n$

Write as $x+iy$ the following numbers:

(1) $\sqrt[4]{16}$
(2) $\sqrt[4]{-1}$
(3) $\sqrt[4]{i}$
(4) $\sqrt[4]{2-i\sqrt{12}}$