Name: **Perm number:**

Final exam – take-home part

Due date: Monday, June 8th

(1) **(25 points)** A **partial order** is a binary relation that is reflexive, antisymmetric, and transitive. Check if the following relations are partial orders:
- $aRb \Leftrightarrow a|b$ defined in the set $\mathbb{Z}$
- $aRb \Leftrightarrow a \leq b$ defined in the set $\mathbb{N}$
- $ARB \Leftrightarrow A \subset B$ defined in the set $P(X)$ for some nonempty set X
- $aRb \Leftrightarrow a \leq b \leq a+1$ defined in the set $\mathbb{Z}$
- $(a,b)R(c,d) \Leftrightarrow (a = c \wedge b < d) \vee (a < c \wedge b = d)$ defined in the set $\mathbb{N} \times \mathbb{N}$

(2) **(25 points)** For a set X with a relation of partial order R the element $x \in X$ is called the **least element of** X if, for all $y \in X$, xRy. Prove that in a partially ordered set there exists at most one least element.

(3) **(25 points)** A **linear order** is a binary operation that is antisymmetric, transitive, and total. A **well-order** relation is a linear order with the property that every non-empty subset S has a least element. Check if the following sets are linearly or well- ordered:
- $\mathbb{Z}$
- $\mathbb{Q}$
- $\mathbb{R}$
- $\{(\frac{1}{n}, 1] : n \in \mathbb{N}\}$
- $P(X)$ for any set X

(4) **(25 points)** Suppose every nonempty subset of a partially ordered set has a least element. Does it follow that this set is well-ordered?