Practice problems for the final

(1) **Line integrals.** Evaluate the line integral, where C is the given curve:
  (a) $\int_C y\mathrm{d}s$, $C: x = t^2, y = t$, $0 \le t \le 2$
  (b) $\int_C (y/x)\mathrm{d}s$, $C: x = t^4, y = t^3$, $1/2 \le t \le 1$
  (c) $\int_C xy^4\mathrm{d}s$, C is the right half of the circle $x^2 + y^2 = 16$
  (d) $\int_C ye^x\mathrm{d}s$, C is the line segment joining $(1, 2)$ to $(4, 7)$
  (e) $\int_C (xy + \ln x)\mathrm{d}s$, C is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$
  (f) $\int_C xe^y\mathrm{d}s$, C is the arc of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$
  (g) $\int_C xy^3\mathrm{d}s$, $C: x = 4\sin t, y = \cos t, z = 3t$, $0 \le t \le \pi/2$
  (h) $\int_C x^2z\mathrm{d}s$, C is the line segment from $(0, 6, -1)$ to $(4, 1, 5)$
  (i) $\int_C xe^{yz}\mathrm{d}s$, C is the line segment from $(0, 0, 0)$ to $(1, 2, 3)$
  (j) $\int_C (2x + 9z)\mathrm{d}s$, $C: x = t, y = t^2, z = t^3$, $0 \le t \le 1$

(2) **Line integrals of vector fields.** Evaluate the line integral, where C is the given curve:
  (a) $\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{x}$, $\mathbf{F}(x, y) = (x^2y^3, -y\sqrt{x})$, C is given by the vector function $\mathbf{x}(t) = (t^2, -t^3)$, $0 \le t \le 1$
  (b) $\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{x}$, $\mathbf{F}(x, y, z) = (yz, xz, xy)$, C is given by the vector function $\mathbf{x}(t) = (t, t^2, t^3)$, $0 \le t \le 2$
  (c) $\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{x}$, $\mathbf{F}(x, y, z) = (\sin x, \cos y, xz)$, C is given by the vector function $\mathbf{x}(t) = (t^3, -t^2, t)$, $0 \le t \le 1$
  (d) $\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{x}$, $\mathbf{F}(x, y, z) = (z, y, -x)$, C is given by the vector function $\mathbf{x}(t) = (t, \sin t, \cos t)$, $0 \le t \le \pi$
  (e) $\int_C xy\mathrm{d}x + (x - y)\mathrm{d}y$, C consists of line segments from $(0, 0)$ to $(2, 0)$, and from $(2, 0)$ to $(3, 2)$
  (f) $\int_C (xy + \ln x)\mathrm{d}y$, C is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$
  (g) $\int_C xe^y\mathrm{d}x$, C is the arc of the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$
  (h) $\int_C x^2y\sqrt{z}\mathrm{d}z$, $C: x = t^3, y = t, z = t^2$, $0 \le t \le 1$
  (i) $\int_C z\mathrm{d}x + x\mathrm{d}y + y\mathrm{d}z$, $C: x = t^3, y = t^3, z = t^2$, $0 \le t \le 1$
  (j) $\int_C (x + yz)\mathrm{d}x + 2x\mathrm{d}y + xyz\mathrm{d}z$, C consists of line segments from $(1, 0, 1)$ to $(2, 3, 1)$, and from $(2, 3, 1)$ to $(2, 5, 2)$
  (k) $\int_C x^2\mathrm{d}x + y^2\mathrm{d}y + z^2\mathrm{d}z$, C consists of line segments from $(0, 0, 0)$ to $(1, 2, -1)$, and from $(1, 2, -1)$ to $(3, 2, 0)$

(3) **The fundamental theorem of calculus for line integrals.** Find a function f such that $\mathbf{F} = \nabla f$, and use it to evaluate $\int_C \mathbf{F}\cdot \mathrm{d}\mathbf{x}$ along the given curve C
  (a) $\mathbf{F}(x, y) = (y, x + 2y)$, C is the upper semicircle that starts at $(0, 1)$ and ends at $(2, 1)$
  (b) $\mathbf{F}(x, y) = (\frac{y^2}{1+x^2}, 2y)$, $C: x = t^2, y = 2t$, $0 \le t \le 1$
  (c) $\mathbf{F}(x, y) = (x^3y^4, x^4y^3)$, $C: x = \sqrt{t}, y = 1 + t^3$, $0 \le t \le 1$
  (d) $\mathbf{F}(x, y, z) = (2xz + y^2, 2xy, x^2 + 3z^2)$, $C: x = t^2, y = t + 1, z = 2t - 1$, $0 \le t \le 1$
  (e) $\mathbf{F}(x, y, z) = (y^2 \cos z, 2xy \cos z, -xy^2 \sin z)$, $C: x = t^2, y = \sin t, z = t$, $0 \le t \le \pi$
  (f) $\mathbf{F}(x, y, z) = (e^y, xe^y, (z + 1)e^z)$, $C: x = t, y = t^2, z = t^3$, $0 \le t \le 1$

(4) **Double integrals.** Evaluate the double integral.
  (a) $\iint_D x^3y^2\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 0 \le x \le 2, -x \le y \le x\}$
  (b) $\iint_D \frac{4y}{x^3+2}\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 1 \le x \le 2, 0 \le y \le 2x\}$
  (c) $\iint_D \frac{2y}{x^2+1}\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$
  (d) $\iint_D e^{y^2}\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 0 \le y \le 1, 0 \le x \le y\}$
  (e) $\iint_D e^{x/y}\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 1 \le y \le 2, y \le x \le y^3\}$
  (f) $\iint_D x\sqrt{y^2 - x^2}\mathrm{d}x\mathrm{d}y$, $D = \{(x, y) : 0 \le y \le 1, 0 \le x \le y\}$
  (g) $\iint_D x \cos y\mathrm{d}x\mathrm{d}y$, D is bounded by $y = 0$, $y = x^2$, $x = 1$
  (h) $\iint_D (x + y)\mathrm{d}x\mathrm{d}y$, D is bounded by $y = \sqrt{x}$, $y = x^2$
  (i) $\iint_D y^3\mathrm{d}x\mathrm{d}y$, D is the triangular region with vertices $(0, 2)$, $(1, 1)$, and $(3, 2)$
  (j) $\iint_D xy^2\mathrm{d}x\mathrm{d}y$, D is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$
  (k) $\iint_D (2x - y)\mathrm{d}x\mathrm{d}y$, D is bounded by the circle with center at the origin and radius 2
  (l) $\iint_D 2xy\mathrm{d}x\mathrm{d}y$, D is the triangular region with vertices $(0, 0)$, $(1, 2)$, and $(0, 3)$

(5) **Triple integrals.** Evaluate the triple integral.
  (a) $\iiint_E 2x\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where $E = \{(x, y, z) : 0 \le y \le 2, 0 \le x \le \sqrt{4 - y^2}, 0 \le z \le y\}$
  (b) $\iiint_E yz \cos(x^5)\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where $E = \{(x, y, z) : 0 \le x \le 1, 0 \le y \le x, x \le z \le 2x\}$
  (c) $\iiint_E 6xy\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E lies under the plane $z = 1 + x + y$, and above the region in the $xy-$plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$
  (d) $\iiint_E y\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $2x + 2y + z = 4$
  (e) $\iiint_E xy\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$
  (f) $\iiint_E xz\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is the solid tetrahedron with vertices $(0, 0, 0)$, $(0, 1, 0)$, $(1, 1, 0)$, and $(0, 1, 1)$
  (g) $\iiint_E x^2e^y\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $z = 0$, $x = 1$, and $x = -1$
  (h) $\iiint_E (x + 2y)\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is bounded by the parabolic cylinder $y = x^2$ and the planes $x = z$, $x = y$, and $z = 0$

(i) $\iiint_E x \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$
(j) $\iiint_E z \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where E is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant

(6) **Green's theorem.** Use Green's Theorem to evaluate the line integrals along the given curves oriented counterclockwise.
(a) $\int_C e^y \mathrm{d}x + 2xe^y \mathrm{d}y$, C is the square with sides $x = 0$, $x = 1$, $y = 0$, and $y = 1$
(b) $\int_C x^2y^2 \mathrm{d}x + 4xy^3 \mathrm{d}y$, C is the triangle with vertices $(0, 0)$, $(1, 3)$, and $(0, 3)$
(c) $\int_C (y + e^{\sqrt{x}})\mathrm{d}x + (2x + \cos y^2)\mathrm{d}y$, C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$
(d) $\int_C xe^{-2x} \mathrm{d}x + (x^4 + 2x^2y^2)\mathrm{d}y$, C is the boundary of the region between the circles $x^2 + y^2 = 1$, and $x^2 + y^2 = 4$
(e) $\int_C y^3 \mathrm{d}x - x^3 \mathrm{d}y$, C is the circle $x^2 + y^2 = 4$
(f) $\int_C \sin y \mathrm{d}x + x \cos y \mathrm{d}y$, C is the ellipse $x^2 + xy + y^2 = 1$