Practice test 1

(1) **Partial derivatives. Tangent planes.** Find an equation of the tangent plane to the given surface at the specified point.
  (a) $z = 4x^2 - y^2 + 2z$, $(-1, 2, 4)$
  (b) $z = 9x^2 + y^2 + 6x - 3y + 5$, $(1, 2, 18)$
  (c) $z = \sqrt{4 - x^2 - 2y^2}$, $(1, -1, 1)$
  (d) $z = y \ln x$, $(1, 4, 0)$
  (e) $z = y \cos(x - y)$, $(2, 2, 2)$
  (f) $z = e^{x^2 - y^2}$, $(1, -1, 1)$

(2) **Linearization. Systems of non-linear differential equations.** Find approximate values of $x(0.02)$ and $y(0.05)$, if $x(0) = 1$, $y(0) = 2$, and
  (a) $x' = 1 - 2xy$, $y' = 2xy - y$
  (b) $x' = x^2 - y^2 - 1$, $y' = 2y$
  (c) $x' = y - x^2 + 2$, $y' = x^2 - xy$
  (d) $x' = 2x - y^2$, $y' = -y + xy$
  (e) $x' = -3x + y^2 + 2$, $y' = x^2 - y^2$
  (f) $x' = xy - 3y - 4$, $y' = y^2 - x^2$
  (g) $x' = -2xy$, $y' = y - x + xy - y^3$
  (h) $x' = x(1 - x^2 - 3y^2)$, $y' = y(3 - x^2 - 3y^2)$
  (i) $x' = x(10 - x - \frac{1}{2}y)$, $y' = y(16 - y - x)$
  (j) $x' = -2x + y + 10$, $y' = 2x - y - 15\frac{y}{y+5}$

(3) **Chain Rule.** Use the Chain Rule to find the indicated partial derivatives:
  (a) $z = x^2 + xy^3$, $x = uv^2 + w^3$, $y = u + ve^w$, $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$, when $u = 2$, $v = 1$, $w = 0$
  (b) $u = \sqrt{r^2 + s^2}$, $r = y + x \cos t$, $s = x + y \sin t$, $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}$, when $x = 1$, $y = 2$, $t = 0$
  (c) $R = \ln(u^2 + v^2 + w^2)$, $u = x + 2y$, $v = 2x - y$, $w = 2xy$, $\frac{\partial R}{\partial x}, \frac{\partial R}{\partial y}$, when $x = y = 1$
  (d) $M = xe^{y - z^2}$, $x = 2uv$, $y = u - v$, $z = u + v$, $\frac{\partial M}{\partial u}, \frac{\partial M}{\partial v}$, when $u = 3$, $v = -1$
  (e) $u = x^2 + yz$, $x = pr \cos\theta$, $y = pr \sin\theta$, $z = p + r$, $\frac{\partial u}{\partial p}, \frac{\partial u}{\partial r}, \frac{\partial u}{\partial \theta}$, when $p - 2$, $r = 3$, $\theta = 0$
  (f) $Y = w \tan^{-1}(uv)$, $u = r + s$, $v = s + t$, $w = t + r$, $\frac{\partial Y}{\partial r}, \frac{\partial Y}{\partial s}, \frac{\partial Y}{\partial t}$, when $r = 1$, $s = 0$, $t = 1$

(4) **Maximas and minimas.**
  (a) Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$.
  (b) Find the shortest distance from the point $(2, 1, -1)$ to the plane $x + y - z = 1$.
  (c) Find the points on the surface $z^2 = xy + 1$ that are closest to the origin.
  (d) Find the points on the surface $x^2y^2z^2 = 1$ that are closest to the origin.
  (e) Find three positive numbers whose sum is 100 and whose product is a maximum.
  (f) Find three positive numbers x, y, and z whose sum is 100 and such that $x^a y^b z^c$ is a maximum.
  (g) Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the ellipsoid

$$9x^2 + 36y^2 + 4z^2 = 36$$

  (h) Solve the above problem for a general ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

  (i) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.
  (j) Find the dimensions of the rectangular box with largest volume if the total surface area is given as 64 cm^2.

(5) **Constrained maximas and minimas.** Find the maximum and minimum values of the function subject to the given constraint:
  (a) $f(x, y) = x^2 - y^2$, $x^2 + y^2 = 1$
  (b) $f(x, y) = 4x + 6y$, $x^2 + y^2 = 13$
  (c) $f(x, y) = x^2y$, $x^2 + 2y^2 = 6$
  (d) $f(x, y) = x^2 + y^2$, $x^4 + y^4 = 1$
  (e) $f(x, y, z) = 2x + 6y + 10z$, $x^2 + y^2 + z^2 = 35$
  (f) $f(x, y, z) = 8x - 4z$, $x^2 + 10y^2 + z^2 = 5$
  (g) $f(x, y, z) = xyz$, $x^2 + 2y^2 + 3z^2 = 6$
  (h) $f(x, y, z) = x^2y^2z^2$, $x^2 + y^2 + z^2 = 1$
  (i) $f(x, y, z, t) = x + y + z + t$, $x^2 + y^2 + z^2 + t^2 = 1$
  (j) $f(x_1, x_2, \ldots, x_n) = x_1 + x_1 + \ldots + x_n$, $x_1^2 + x_2^2 + \ldots + x_n^2 = 1$