Name: **Perm number:**

Midterm – take-home part

Due date: Monday, May 18th

(1) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function, let $g_1, \ldots, g_m : \mathbb{R}^n \to \mathbb{R}$ be other continuously differentiable functions. Explain why the maximum of the function f subject to the conditions

$$g_1(x_1, \ldots, x_n) = 0, g_2(x_1, \ldots, x_n) = 0, \ldots, g_m(x_1, \ldots, x_n) = 0$$

occurs at the point $(x_1, \ldots, x_n)$ such that

$$\nabla f(x_1, \ldots, x_n) = \lambda_1 \nabla g_1(x_1, \ldots, x_n) + \lambda_2 \nabla g_2(x_1, \ldots, x_n) + \ldots + \lambda_m \nabla g_m(x_1, \ldots, x_n)$$

(hint: start with the case when $n = 3$, $m = 2$)

(2) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function, let $g_1, \ldots, g_m : \mathbb{R}^n \to \mathbb{R}$ be other continuously differentiable functions. Let

$$H(x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_m) = f(x_1, \ldots, x_n) - \lambda_1 g_1(x_1, \ldots, x_n) - \ldots - \lambda_m g_m(x_1, \ldots, x_n)$$

Prove that $\nabla H(x_1, \ldots, x_n, \lambda_1, \ldots, \lambda_m) = 0$ if and only if

$$\nabla f(x_1, \ldots, x_n) = \lambda_1 \nabla g_1(x_1, \ldots, x_n) + \lambda_2 \nabla g_2(x_1, \ldots, x_n) + \ldots + \lambda_m \nabla g_m(x_1, \ldots, x_n)$$

and

$$g_1(x_1, \ldots, x_n) = 0, g_2(x_1, \ldots, x_n) = 0, \ldots, g_m(x_1, \ldots, x_n) = 0$$

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(3) Find the maximum and minimum values of the function $f(x, y, z) = 3x - y - 3z$ subject to the constraints $x + y - z = 0$ and $x^2 + 2z^2 = 1$.

(4) Find the maximum and minimum values of the function $f(x, y, z) = xyz$ subject to the constraints $x^2 + y^2 = 1$ and $x - z = 0$.

(5) Find the maximum and minimum values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x + y + z = 1$ and $x^2 + y^2 - z^2 = 0$.