Math 5AI First Midterm Topics

The nature of solutions to homogeneous and non-homogenous linear ODEs.

The solutions in the constant coefficient case.

What complex numbers have to do with real solutions to ODE.

Finding a basis for the solutions in the homogeneous case.

Use the terminology of linear independence and span correctly.

Finding solutions that satisfy initial conditions.

What the Wronskian has to do with it all.

Finding Power Series Solutions to Linear ODE.

Math 5AI Midterm Review Questions

1. (i) Find a homogeneous differential equation for which $y_1 = e^{(1+i)x}$, $y_2 = e^{(1-i)x}$, and $y_3 = e^x$ form a basis of solutions.
(ii) Find a basis of solutions which are real-valued functions. NOTE: You should be able to do this in a few lines using what you know–no long computations are required.

2. Find a solution to the ODE

$$y'' + 4y = \sin 2t$$

For which $y(0) = 1$ and $y'(0) = 1$.

3. (i) Find all solutions to

$$x^2y'' + 5xy' + 4y = 0$$

(ii) Find all solutions to

$$x^2y'' + 5xy' + 4y = 9x$$

4. Find two power series solutions to

$$y'' + xy = 0$$

and show they are linearly independent. You may assume that the power series you obtain converge.

5. Suppose you know that the ODE

$$y''' + p(x)y'' + q(x)y' + r(x)y = 0$$

has a 3-dimensional solution space. Suppose that $y = f(x)$ is a solution to

$$y''' + p(x)y'' + q(x)y' + r(x)y = x$$

Explain, using $f(x)$ and $g(x)$ how to find a solution $h(x)$ of

$$y''' + p(x)y'' + q(x)y' + r(x)y = x$$

for which $y = h(x)$ with $h(0) = 1$ but $h'(0) = 0$ and $h''(0) = 0$. NOTE: Do not try to calculate a solution– just *explain why it exists*.

1. Find all solutions to $y'' + y = x^2$.

2. Consider the ODE

$$y''' - y'' + y' - y = 0.$$

(a) What are all of its solutions?
(b) What are all of its solutions for which $y(0) = 0$, $y'(0) = 1$, and $y''(0) = 0$?
(b) What are all of its solutions for which $y(0) = 0$, $y'(0) = 0$, and $y''(0) = 0$?

3. Show how to derive a power series solution to $y'' + 2y = 0$ for which $y(0) = 1$ and $y'(0) = 0$. List terms up to x^4.