Name: **Perm number:**

Midterm – in-class part

Time: 75 minutes

1. **(25 points)**
   - Find all real solutions to $y'' + y = x^2$.
   - Find all real solutions to $y'' + y = x^2$ for which $y(0) = 0$ and $y'(0) = 0$.

2. **(25 points)** Consider the equation

$$y''' - y'' + y' - y = 0$$

- What are all of its real solutions?
- What are all of its real solutions for which $y(0) = 0$, $y'(0) = 1$, and $y''(0) = 0$?

3. **(25 points)** Show how to derive a power series solution to $y'' + 2y = 0$ for which $y(0) = 1$ and $y'(0) = 0$. List terms up to x^4.

4. **(25 points)** Suppose the second order linear ODE

$$y'' + p(x)y' + q(x)y = 0$$

has two solutions $f(x)$ and $g(x)$ and you know that the Wronskian

$$W(f(x), g(x)) > 0$$

for all x. Using $f(x)$ and $g(x)$ explain how to find a solution $h(x)$ of

$$y'' + p(x)y' + q(x)y = 0$$

for which $h(1) = 3$ and $h'(1) = 7$.