Introductory project – the May model

The May model has beeen proposed by the contemporary ecologist, R.M.May, to incorporate certain assumptions about the encounters between predators (foxes) and their prey (rabbits). Let y be the number of foxes and let x be the number of rabbits divided by 100 – we are thus measuring rabbits in units of what we could call centirabbits.

In his model, May makes the following assumptions:

- In the absence of foxes, the rabbits grow logistically, i.e. the population of rabbits can be modelled by the equation
$$(100 \cdot x)' = k \cdot 100 \cdot x(1 - \frac{100 \cdot x}{b}),$$
where the coefficient k is called the natural growth rate, and the number b is the carrying capacity of the ecosystem we are considering.
- The number of rabbits a single fox eats in a given time period is a function $D(x)$ of the number of rabbits available. $D(x)$ varies from 0 if there are no rabbits available to some value c (the saturation value) if there is an unlimited supply of rabbits. The total number of rabbits consumed in the given time period will thus be $D(x) \cdot y$.
- The fox population is governed by the logistic equation, and the carrying capacity is proportional to the number of rabbits.

(1) Explain why
$$D(x) = \frac{cx}{x + d}$$
(where d is some constant) might be a reasonable model for the function $D(x)$. Include a sketch of the graph of D in your discussion. What is the role of the parameter d? That is, what feature of rabbit-fox interactions is reflected by making d smaller or larger?

(2) Write down a system of equations that incorporates May's assumptions.

(3) Assume you begin with 2000 rabbits and 10 foxes. What does May's model predict will happen to the rabbits and foxes over time? Use some random but reasonable numbers for parameters such as the carrying capacity etc.

(4) Using the same parameters describe what happens if you begin with 2000 rabbits and 20 foxes, with 1000 rabbits and 10 foxes, with 1000 rabbits and 20 foxes.

(5) Using 2000 rabbits and 20 foxes as the initial values, let's see how the behavior of the solutions is affected by changing the values of the parameter c, the saturation value for the number of rabbits a single fox can eat in a month. The solutions will undergo a qualitative change somewhere between two values of c (find them!). Describe this change. Can you pinpoint the crucial value of c more closely? This phenomenon is an example of Hopf bifurcation that we will discuss later.