Math 3CI Implicit Functions and DiffyQ

1. Recall that the circle of radius r centered at (a, b) is described in the plane by the equation $(x-a)^2 + (y-b)^2 = r^2$.
(a) This equation does **not** define a function. Why?
(b) But it can be used to define several functions *implicitly*. What does this mean?
(c) Use the chain rule to find the slope of a tangent line at a point (r, s) of the graph of one of these implicit functions. You may use the chain rule–but you are not allowed to use any dx magic!

2. The equation $y^2 = x^3 - x$ has as graph a famous elliptic curve.
(a) Give a formula for the slope of a tangent line to any point (r, s) of this curve.
(b) Where are the tangents vertical?
(c) Suppose a function $f(x)$ is implicitly defined by $y^2 = x^3 - x$. What differential equation does $f(x)$ satisfy? Again, you may use the chain rule–but you are not allowed to use any dx magic!
(d) What does the slope field for the differential equation in (c) look like? Plot $y^2 = x^3 - x$ on the same plane.

3. Use the ideas you developed in problem 2. to find some solutions to the following differential equations. You may describe the solution as being an implicitly defined function. Use the chain rule in your explanations.
(a) $y' = \frac{x+1}{y-1}$
(b) $y' = x(1+y)^2$
(c) $y' = e^{x+y}$