Math 3CI Introducing ODE Techniques

In your study of the first rabbit population problem you determined that since the rabbit population function $R(t)$ has a constant per capita growth rate of 0.1 rabbits per month per rabbit, that the following equation must be true:

$$\frac{dR}{dt}(t) = 0.1R(t).$$

This is called an ordinary differential equation. You then were able to recall from Calculus that possible $R(t)$ functions are $R(t) = Ke^{0.1t}$ where K is some constant. This shows one way of solving a differential equation. If you know enough about some functions then you can solve the equation. Finding the value of K is something you may need to discuss in your group, however.

Now, what happens with other equations? Can you always guess, or can there be some strategies? Well, there are other strategies of course, but for now, you can play around with functions and try to find solutions.

1. Play with exponential functions and find at least two solutions to

$$\frac{d^2y}{dt^2}(t) - \frac{dy}{dt}(t) - 2y(t) = 0.$$

2. Play with some other functions you studied in calculus and find some solutions to

$$\frac{d^2y}{dt^2}(t) + 4y(t) = 0.$$

3. Play more with modified exponential functions and find solutions to

$$\frac{dy}{dt}(t) - 2ty(t) = 0.$$

4. Find at least two solutions to

$$\frac{d^2y}{dt^2}(t) - 4\frac{dy}{dt}(t) + 4y(t) = 0.$$

5. Draw a graph of what several solutions to

$$\frac{dy}{dt}(t) - (y(t))^2 = 0.$$

could look like, but don't try to directly find a formula. You will have to make use of the slope-field idea developed in the rabbit with munchies problem. Do your graphs look like any functions you have seen before? If so, try to test their formulas to see if they could be solutions.

6. You know a solution to

$$\frac{dy}{dt}(t) - y(t) = 0 \quad \text{with} \quad y(0) = 1.$$

By how do you know the function $y = e^t$ really exists? One way is by knowing that the differential equation has a solution! Recall that we could approximate solutions to ODEs by iterative approximations, and that as the step size gets smaller, the approximations get better and better. Show equation above has a solution for $t \geq 0$ by considering approximations to $y(x)$ where the step sizes are 1, 1/2, 1/4, 1/8 ... Don't try to find the function in one sweep. Instead calculate the approximations and look for a pattern in your result. Then extend it. Play with these ideas. We will keep talking about it.