Math 3CI Foxes and Rabbits

We have seen how difference equations and differential equations arise from simple assumptions about population growth. Here we will consider how two populations, one eating the other, can lead to more interesting models of population interaction. The ideas we look at form the foundations for modern population ecology, and are due to Italian physicist-turned ecologist Vito Volterra and the American statistician Alfred Lotka (who had a lifelong interest in the biological sciences) during the mid 1920's.

1. Your task is to build two equations (a system of differential and difference equations) built upon the assumptions given below (R = rabbit population, F = fox population).
(a) In the absence of foxes, the rabbit population grows at a constant per capita rate.
(b) The population of rabbits declines at a rate proportional to the product RF . (This assumption combines the death rate and the "those eaten" rate into one simplifying assumption.) Why might this be reasonable?
(c) In the absence of rabbits, the foxes die off at a rate proportional to the number of foxes present.
(d) The fox population grows at a rate proportional to the product of RF. Why might this be reasonable?

2. Now that you have created your equations, input the following constants for your Island population.
(a) has the constant 0.1 rabbit per month per rabbit
(b) has the constant 0.005 rabbit per month per rabbit-fox
(c) has the constant 0.04 foxes per month per fox
(d) has the constant 0.00004 foxes per month per rabbit-fox
(A) Write down your equations (be care of the sign: (a) gives growth rate, while (c) gives a die-off rate.)
(B) Now, a month by month computation would drive you crazy (and many students in this course are really getting sick of such computation) so instead we will use calculators/computers to study solutions. HOWEVER, before you plot solutions using a calculator or computer, I want you to study selected values of possible pairs (R,F) and try to characterize which values correspond to the four possibilities of rabbits increasing/decreasing, and

foxes increasing/decreasing.
(C) Suppose you start with 10 foxes and 2000 rabbits. What happens after 200 months? Make a crude sketch of what you think could happen *based upon the information you generated in part (B)*. You will be making crude estimates using the key ideas in the model – that is OK because you know a computer calculation can be done subsequently. NOTE: your need to think in advance about how to use your time efficiently – get some bounds quickly, and don't try to compute all values. Remember our experience with the slope fields on Monday? If you don't make crude estimates to set up reasonable axes you can spend all your time looking at insignificant values for the problem. See what you can estimate in a short period of time. Do not use a computer or a calculator for this, and do not do a month by month calculation by hand either.
(D) Now if you like, using technology, make some specific calculations. But these are not the object here.