Practice Test 2

1. Suppose the second order linear ODE
$$y'' + p(x)y' + q(x)y = 0$$
has two solutions $f(x)$ and $g(x)$ and you know that the Wronskian
$$W(f(x), g(x)) > 0$$
for all x. Using $f(x)$ and $g(x)$ explain how to find a solution $h(x)$ of
$$y'' + p(x)y' + q(x)y = 0$$
for which $h(1) = 3$ and $h'(1) = 7$.

2. Prove the following properties of matrix exponential:
   (a) $e^{aX}e^{bX} = e^{(a+b)X}$
   (b) $e^{X}e^{-X} = I$
   (c) if $AB = BA$, then $e^{A}e^{B} = e^{A+B}$
   (d) $\det e^{X} = e^{trX}$
   (e) $e^{A^T} = (e^{A})^T$

3. (a) Supose that
$$A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}.$$
   Use the definition of what it means to exponentiate a matrix to calculate the matrix of functions e^{At}.

   (b) Use your answer to (a) to help you calculate e^{Bt} where
$$B = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix}.$$

   (c) Consdier the system of ODE's given by
$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$
   With your answer to (b) you can easiy write down the general solution to this system Do so.

   (d) Now, in the system given in (c), show that $y_1'' - 6y_1' = -9y_1$ and relate the solution to this second order ODE to the to solution for the system of equations. (Hint: note that as $y_1' = 3y_1 + 2y_2$ we can differentiate and find that $y_1'' = (3y_1 + 2y_2)' = 3y_1' + 2y_2' = 3(3y_1 + 2y_2) + 2(3y_2) = 9y_1 + 12y_2$.)