Practice test 3

1. **Fundamental theorem of calculus**
   - Find the intervals where the function $f(x) = \int_0^x t^2 dt$ is increasing.
   - Find the intervals where the function $f(x) = \int_\pi^x \sin t dt$ is decreasing.
   - Find local minimas and maximas of the function $f(x) = \int_x^{2x} t^3 dt$.
   - Find the intervals where the function $f(x) = \int_x^{4x} \cos s ds$ is concave up, and the intervals where it is concave down.
   - Find points of inflection of the function $f(x) = \int_{\sin x}^{x} \cot s ds$.
2. **Areas between curves**
   - Find the area bounded between the curves $y = t^2$ and $y = t$.
   - Find the area bounded between the curves $y = x^2$ and $y = 1$.
   - Find the area bounded between the curves $y = 2t - t^2$ and $y = t$.
   - Find the area bounded between the curves $y = 6 - x^2$ and $y = 4 - x$.
   - Find the area bounded between the curves $y = x^2$ and $y = 2 - x^2$.
   - Find the area bounded between the curves $y = x^3 + 1$ and $y = x + 1$.
   - Find the area bounded between the curves $y = 1/2x^3$ and $y = 2x$.
   - Find the area bounded between the curves $y = (1/9)(x^3 + x^2) + 3$ and $y = x + (1/9)x^2 + 3$.
   - Find the area bounded between the curves $y = 2x^2 - x^3 + 3$ and $y = 5 - x$.
   - Find the area bounded between the curves $y = x^3 - 2x^2 + 4$ and $y = x + 2$.
3. **Volumes**
   - Find the volume of the solid of revolution formed when the region bounded between $x = 1$, $x = 3$, $y = x^2$ and $y = 0$ is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between $x = 0$, $x = \pi$, $y = \sin(x)$ and $y = 0$ is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between $x = \pi/4$, $x = \pi/2$, $y = \sin(x)$ and $y = 1/2$ is revolved vertically around the x-axis.
   - Find the volume of the solid of revolution formed when the region bounded between $x = 2$, $x = 5/2$, $y = x^2$ and $y = 2$ is revolved vertically around the line $y = 1$.
   - Find the volume of the solid of revolution formed when the region bounded between $x = 1$, $x = 2$, $y = e^{x/3}$ and $y = 1$ is revolved vertically around the line $y = -1$.
   - Find the volume of the solid of revolution formed when the region bounded between $y = x + 5$ and $y = x^2 + 3$ is revolved vertically around the line $y = 2$.
   - Find the volume of the solid of revolution formed when the region bounded between $y = x$ and $y = x^2$ is revolved horizontally around the y-axis.
   - Find the volume of the solid of revolution formed when the region bounded between $y = 2x - 6$ and $y = (x - 3)^2$ is revolved horizontally around the line $x = 1$.
   - Find the volume of the solid of revolution formed when the region bounded between $y = 6 - 2x$ and $y = (x - 2)^2 + 2$ is revolved vertically around the line $y = -1$.
   - Find the volume of the solid of revolution formed when the region bounded between $y = x$ and $y = x^2$ is revolved horizontally around the line $x = -2$.
   - Find the volume of the solid of revolution formed when the region below the curve $y = x^2$, above the x-axis, and between $x = 0$ and $x = 1$ is revolved around the y-axis.
   - Find the volume of the solid of revolution formed when the region below the curve $y = \cos(x)$, above the x-axis, and between $x = 0$ and $x = \pi/2$ is revolved around the y-axis.
   - Find the volume of the solid of revolution formed when the region bounded between $y = 3x + 2$ and $y = x^2 + 2$ is revolved horizontally around the y-axis.

- Find the volume of the solid of revolution formed when the region bounded between $y = x + 2$ and $y = x^2$ is revolved horizontally around the line $x = -2$.
- Find the volume of the solid of revolution formed when the region bounded between $y = x - 1$ and $y = (x - 1)^2$ is revolved vertically around the line $y = -1$.
- Find the volume of a right circular cone with height h and base radius r.
- Find the volume of a frustrum of a right circular cone with height h, lower base radius R, and top radius r.
- Find the volume of a cap of sphere with radius r and height h.
- Find the volume of a frustrum of a pyramid with square base of side b, square top of side a, and height h.
- Find the volume of a pyramid with height h and rectangular base with dimensions b and $2b$.
- Find the volume of a pyramid with height h and base an equilateral triangle with side a.
- Find the volume of a tetrahedron with three mutually perpendicular faces and three mutually mutually perpendicular edges with lengths 3 cm, 4 cm, and 5 cm.
- Find the volume of the solid S whose base is a circular disk with radius r, and parallel cross-sections perpendicular to the base are squares.
- Find the volume of the solid S whose base is a circular disk with radius r, and parallel cross-sections perpendicular to the base are equilateral triangles.
- Find the volume of the solid S whose base is an elliptical region bounded by the curve $9x^2 + 4y^2 = 36$, and parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse in the base.
- Find the volume of the solid S whose base is a parabolical region bounded by the curves $y = x^2$ and $y = 1$, and parallel cross-sections perpendicular to the base are equilateral triangles.
- Find the volume of the solid S whose base is a parabolical region bounded by the curves $y = x^2$ and $y = 1$, and parallel cross-sections perpendicular to the base are half-circles.

4. **Work**

- A force of 10 lb is required to hold a spring stretched 4 inches beyond its natural length. How much work (in ft-lb) is done in stretching it from its natural length to 6 inches beyond its natural length?
- A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work (in J) is required to stretch it from 20 cm to 25 cm?
- Suppose that 2 J of work are needed to stretch a spring from its natural length of 30 cm to a length of 42 cm. How much work (in J) is needed to stretch it from 35 cm to 40 cm?
- Suppose that 2 J of work are needed to stretch a spring from its natural length of 30 cm to a length of 42 cm. How far (in cm) beyond its natural length will a force of 30 N keep this spring stretched?
- If 6 J of work are needed to stretch a spring from 10 cm to 12 cm and another 10 J are needed to stretch it from 12 cm to 14 cm, what is the natural length (in cm) of the spring?
- A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done (in ft-lb) in pulling the rope to the top of the building? How much work is done (in ft-lb) in pulling half the rope to the top of the building?
- A chain lying on the ground is 10 m long and its mass is 80 kg. How much work (in J) is required to raise one end of the chain to a height of 6 m?
- A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done (in ft-lb).
- A bucket that weighs 4 lb and a rope of negligible weight are used to draw water from a well that is 80 ft deep. The bucket starts with 40 lb of water and is pulled up at a rate of 2 ft/s, but water leaks out of a hole in the bucket at a rate of 0.2 lb/s. Find the work done (in ft-lb) in pulling the bucket to the top of the well.
- A leaky 10-kg bucket is lifted from the ground to a height of 12 m at a constant speed with a rope that weighs 0.8 kg/m. Initially, the bucket contains 36 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12 m level. How much work is done (in J)?
- A 10-ft chain weighs 25 lb and hangs from a ceiling. Find the work done (in ft-lb) in lifting the lower end of the chain to the ceiling so that it is level with the upper end.
- An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work (in J) needed to pump half of the water out of the aquarium. Use the fact that the density of water is $1000\,kg/m^3$.

- A circular swimming pool has a diameter of 24 ft, the sides are 5 ft high, and the depth of the water is 4 ft. How much work (in ft-lb) is required to pump all of the water out over the side? Use the fact that water weighs $62.5\,lb/ft^3$.
- When gas expands in a cylinder with radius r, the pressure at any given time is a function of the volume: $P = P\,(V)$. The force exerted by the gas on the piston is the product of the pressure and the area: $F = \pi r^2 P$. One can then show that the work done by the gas when the volume expands from volume V_1 to volume V_2 is $W = \int_{V_1}^{V_2} P\,dV$

  In a steam engine the pressure P and volume V of steam satisfy the equation $PV^{1.4} = k$, where k is a constant. (This is true for adiabatic expansion, that is, expansion in which there is no heat transfer between the cylinder and its surroundings.) Use the information given above to calculate the work done (in ft-lb) by the engine during a cycle when the steam starts at a pressure of $160\,lb/in^2$ and a volume of $100\,in^3$ and expands to a volume of $800\,in^3$.
- Newton's Law of Gravitation states that two bodies with masses M and N attract each other with a force $F = G\frac{MN}{r^2}$, where r is the distance between the bodies and G is the gravitational constant.

  Use Newton's Law of Gravitation to compute the work (in J) required to launch a 1000-kg satellite vertically to an orbit 1000 km high. You may assume that Earth's mass is $5.98 \times 10^{24}\,kg$ and is concentrated at its center. Take the radius of the Earth to be $6.37 \times 10^6\,m$ and $G = 6.67 \times 10^{-11}\,N \cdot m^2/kg^2$.

5. **Average value**
   - Find the average value of $f(x) = x$ on the interval [0,1].
   - Find the average value of $f(x) = x^2 + 1$ on the interval [1,3].
   - Find the average value of $f(x) = x^2 - x$ on the interval [0,1].
   - Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$.
   - Find the average value of $f(x) = 3$ on the interval [5,18].

6. **Methods of integration.** Evaluate the following integrals:
   - $\int_0^2 \frac{6t}{(t-3)^2}\,dt$
   - $\int_{-1}^1 \frac{2e^{\arctan(y)}}{1+y^2}\,dy$
   - $\int_1^3 -10r^4 \ln(r)\,dr$
   - $\int_0^4 \frac{-6x+6}{x^2-4x-5}\,dx$
   - $\int \frac{-7x+7}{x^2-4x+5}\,dx$
   - $\int \frac{3x}{x^4+x^2+1}\,dx$
   - $\int_0^{1/2} \frac{-4x}{\sqrt{1-x^2}}\,dx$
   - $\int \frac{4e^{2t}}{1+e^{4t}}\,dt$
   - $\int 4t^3 e^{-2t}\,dt$
   - $\int \frac{12x^2-8}{x^2-2x-8}\,dx$
   - $\int \frac{3x^2-2}{x^3-2x-8}\,dx$
   - $\int_0^5 \frac{15w-5}{w+2}\,dw$
   - $\int 5e^x\sqrt{1+e^x}\,dx$
   - $\int -5x^5 e^{-x^3}\,dx$
   - $\int \frac{6+6e^x}{1-e^x}\,dx$
   - $\int \frac{-4(x+a)}{x^2+a^2}\,dx$
   - $\int \frac{7x^4}{x^{10}+16}\,dx$
   - $\int \frac{-2x^3}{(x+1)^{10}}\,dx$
   - $\int_2^3 \frac{-4u^3-4}{u^3-u^2}\,du$
   - $\int \frac{1x}{x^4+4x^2+3}\,dx$

- $\int \frac{3}{(x-2)(x^2+4)}\, dx$

7. **Length of a curve**
   - Find the arc length of the curve $f(x) = 2x - 1$ over the interval $[0, 3]$.
   - Find the arc length of the curve $f(x) = x^{3/2} - 1$ over the interval $[0, 1]$.
   - Find the arc length of the curve $f(x) = x^4/8 + 1/(4x^2)$ over the interval $[2, 3]$.
   - Find the arc length of the curve $f(x) = 2/3(x-1)^{3/2}$ over the interval $[1, 4]$.
   - Find the arc length of the curve $f(x) = (e^x + e^{-x})/2$ over the interval $[0, 1]$.
   - Find the arc length of the curve $f(x) = ln(cosx)$ over the interval $[0, \pi/4]$.
   - Find the arc length of the curve $f(x) = 1/4x^2 - 1/2lnx$ over the interval $[1, 4]$.
   - Find the arc length of the curve $f(x) = 3/8x^{4/3} - 3/4x^{2/3}$ over the interval $[1, 8]$.
   - Find the arc length of the curve $f(x) = 1/6x^3 + 1/2x^{-1}$ over the interval $[1, 2]$.

8. **Area of a surface of revolution**
   - Find the area of the surface obtained by rotating the curve $y = x^3,\ 0 \le x \le 2$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $9x = y^2 + 18,\ 2 \le x \le 6$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $y = \sqrt{x},\ 4 \le x \le 9$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $y = \cos(2x),\ 0 \le x \le \pi/6$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $y = \cosh(x),\ 0 \le x \le 1$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x},\ \dfrac{1}{2} \le x \le 1$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $x = \frac{1}{3}(y^2+2)^{3/2},\ 1 \le y \le 2$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $x = 1 + 2y^2,\ 1 \le y \le 2$, about the x-axis.
   - Find the area of the surface obtained by rotating the curve $y = \sqrt[3]{x},\ 1 \le y \le 2$, about the y-axis.
   - Find the area of the surface obtained by rotating the curve $y = 1 - x^2,\ 0 \le x \le 1$, about the y-axis.
   - Find the area of the surface obtained by rotating the curve $x = \sqrt{a^2 - y^2},\ 0 \le y \le a/2$, about the y-axis.
   - Find the area of the surface obtained by rotating the curve $x = a\cosh(y/a),\ -a \le y \le a$, about the y-axis.