

Practice test 1

1. Differential equations.

- Find the solution of the differential equation $y'(x) = x$ with the initial condition $y(0) = 4$.
- Find the solution of the differential equation $y'(t) = 3t - 7$ with the initial condition $y(0) = 0$.
- Find the solution of the differential equation $y'(x) = 3x^2$ with the initial condition $y(0) = 4$.
- Find the solution of the differential equation $y'(x) = x^3 - x$ with the initial condition $y(0) = -3$.
- Find the solution of the differential equation $y'(t) = \sin t + 3t^2$ with the initial condition $y(0) = 1$.
- A wanna-be climber drops a carabiner off a cliff, which hits the ground with a speed of $120ft/s$. What is the height of the cliff?
- A car is travelling at $50mi/h$ when the brakes are fully applied, producing a constant deceleration of $22ft/s^2$. What is the distance covered before the car comes to a stop?
- What constant acceleration is required to increase the speed of a car from $30mi/h$ to $50mi/h$ in $5s$?
- A car braked with a constant deceleration of $16ft/s^2$, producing skid marks measuring $200ft$ before coming to stop. How fast was the car travelling when the brakes were first applied?
- A car is travelling at $100km/h$ when the driver sees an accident $80m$ ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup? What if the driver was talking on a cell phone and, as a result, hit the brakes $2s$ later?

2. Definite integrals. Compute these definite integrals:

- $\int_0^2 1 - 2v + v^2 dv$
- $\int_1^3 w^3 - 3w^2 + 4w - 1 dw$
- $\int_0^4 y^2 - y^{1/2} dy$
- $\int_7^7 2y^4 - y^2 dy$
- $\int_4^9 x^{1/2} dx$
- $\int_0^\pi \sin t dt$
- $\int_{-\pi/2}^{\pi/2} \sin u du$
- $\int_0^{\pi/3} 1 - \sec x \tan x dx$
- $\int_{\pi/6}^{\pi/4} 2 \sec^2 t dt$
- $\int_0^{\pi/2} 2x + \cos x dx$

3. Differentiation and the Fundamental Theorem.

- Find the derivative of the functions $f(x)$ defined by the following integral $f(x) = \int_0^x t^2 dt$.
- Find the derivative of the functions $f(x)$ defined by the following integral $f(x) = \int_\pi^x \sin t dt$.
- Find the derivative of the functions $f(x)$ defined by the following integral $f(x) = \int_x^{2x} t^3 dt$
- Find the derivative of the functions $f(x)$ defined by the following integral $f(x) = \int_x^{4x} \cos s ds$
- Find the derivative of the functions $f(x)$ defined by the following integral $f(x) = \int_{\sin x}^x \cot s ds$

4. Integration by substitution. Evaluate

- $\int (2x + 1)^2 dx$
- $\int \sin^2 y \cos y dy$
- $\int (5 + 6x)^{-3} dx$
- $\int \sqrt{x+5} dx$
- $\int 1/\sqrt{5-7x} dx$
- $\int \sec(2t-3) \tan(2t-3) dt$

- $\int x^2(x^3 - 8)^{11} dx$
- $\int x\sqrt{x^2 + 3} dx$
- $\int y^4(3y^5 + 1)^{4/3} dy$
- $\int \sin(x)/\cos^4(x) dx$