Practice test 4

1. **Optimization problems.**
   - 8.13.6 - 8.13.20
2. **The definite integral.** Evaluate the following integrals
   - $\int_{-1}^{3} x^5 dx$
   - $\int_{-2}^{5} 6 dx$
   - $\int_{2}^{8} (4x+3) dx$
   - $\int_{0}^{4} (1+3y-y^2) dy$
   - $\int_{0}^{1} x^{\frac{4}{5}} dx$
   - $\int_{1}^{8} \sqrt[3]{x} dx$
   - $\int_{1}^{2} \frac{3}{t^4} dt$
   - $\int_{-2}^{3} x^{-5} dx$
   - $\int_{-5}^{5} \frac{2}{x^3} dx$
   - $\int_{\pi}^{2\pi} \cos\theta d\theta$
   - $\int_{0}^{2} x(2+x^5) dx$
   - $\int_{1}^{4} \frac{1}{\sqrt{x}} dx$
   - $\int_{0}^{1} (3+x\sqrt{x}) dx$
3. **Antiderivatives and indefinite integrals.** Find f if
   - $f'(x) = \sqrt{x^5} - \frac{4}{\sqrt[5]{x}}$
   - $f'(x) = e^x - \frac{2}{\sqrt{x}}$
   - $f'(t) = 2t - 3\sin t$ and $f(0) = -1$
   - $f'(u) = \frac{u^2+\sqrt{u}}{u}$ and $f(1) = 3$
   - $f''(x) = 1 - 6x + 48x^2$, and $f(0) = 1$, and $f'(0) = 2$
   - $f''(x) = 2x^3 + 3x^2 - 4x + 5$, and $f(0) = 2$, and $f(1) = 0$
4. **The product rule.**
   - 12.2.5 - 12.2.18
5. **Second derivatives.** For the following functions find the intervals of increase and decrease, the local maximum and minimum values, the intervals of concavity, and the inflection points
   - $f(x) = 2x^3 - 3x^2 - 12x$
   - $f(x) = 2 + 3x - x^3$
   - $f(x) = x^4 - 6x^2$
   - $g(x) = 200 + 8x^3 + x^4$
   - $h(x) = 3x^5 - 5x^3 + 3$
   - $h(x) = (x^2-1)^3$
   - $B(x) = 3x^{2/3} - x$
   - $C(x) = x^{1/3}(x+4)$
   - $f(x) = 2\cos x - \cos 2x$, for $0 \le x \le 2\pi$
   - $f(t) = t + \cos t$, for $-2\pi \le t \le 2\pi$
6. **Graphing solutions of differential equations.**

- Sketch the slope field for the equation $y' = y$. Sketch the solution curve for which $y(-1) = 0.2$. What is the long term behavior of this solution?
- Sketch the slope field for the equation $y' = t^2 - y^2$. Sketch the solution curve for which $y(1) = -0.5$. What is the long term behavior of this solution?
- Sketch the slope field for the equation $y' = 3y(.5 - y^2)$. Sketch the solution curve for which $y(0) = 0.5$. What is the long term behavior of this solution?

7. **Exponential decay towards a limiting value.**

   - 13.7.9 - 13.7.12

8. **Logistic equation.**

   - Suppose that a population grows according to a logistic model with carrying capacity 6,000 and $k = 0.0015$ per year. Write the logistic differential equation for these data. If the initial population is 1,000, find the population after 50 years.
   - The Pacific halibut fishery has been modeled by the differential equation

     $$y' = ky(1 - \frac{y}{K})$$

     where $y(t)$ is the biomass in kilograms at time t, the carrying capacity is estimated to be $K = 8 \times 10^7$ kg, and $k = 0.71$ per year. If the initial biomass is 2×10^7 kg, how long will it take for the biomass to reach 4×10^7 kg?
   - One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor. Write a differential equation that is satisfied by y. Solve the equation. A small town has 1,000 inhabitants. At 8 AM, 80 people have heard that a child was abducted by an unidentified driver of a 1965 Lincoln Continental. By noon half the town already knew it. At what time will 90% of the population have heard about children being abducted in black cars?
   - Biologists stocked a lake with 400 fish and estimated the carrying capacity to be 10,000. The number of fish tripled in the first year. How long will it take for the population to increase to 5,000?
   - The population of the world was about 5.3 billions in 1990. Birth rates in 1990s ranged from 35 to 40 million per year and death rates ranged from 15 to 20 million per year. Scientists generally agree that the carrying capacity for the world population is 100 billlion. What is the predicted world population in the years 2100 and 2500? Would you like to live to 120?

9. **Partial derivatives and tangent planes.**

   - 15.1.1 - 15.1.3
   - 15.2.1 - 15.2.5

10. **Maximas and minimas of functions of many variables.**

    - 15.4.1 - 15.4.5