

Practice test 3

1. Differential equations.

- 13.2.6 – 13.2.10
- A wanna-be climber drops a carabiner off a cliff, which hits the ground with a speed of 120ft/s . What is the height of the cliff?
- A car is travelling at 50mi/h when the brakes are fully applied, producing a constant deceleration of 22ft/s^2 . What is the distance covered before the car comes to a stop?
- What constant acceleration is required to increase the speed of a car from 30mi/h to 50mi/h in 5s ?
- A car braked with a constant deceleration of 16ft/s^2 , producing skid marks measuring 200ft before coming to stop. How fast was the car travelling when the brakes were first applied?
- A car is travelling at 100km/h when the driver sees an accident 80m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup? What if the driver was talking on a cell phone and, as a result, hit the brakes 2s later?

2. The growth/decay equation.

- A common inhabitant of human intestines is the bacterium *Escherichia coli*. A cell of this bacterium in a nutrient-broth medium divides into two cells every 20 minutes. The initial population of a culture is 60 cells. Find an expression for the number of cells after t hours. Find the number of cells after 8 hours. When will the population reach 20,000 cells?
- A bacteria culture starts with 500 bacteria and grows at a rate proportional to its size. After 3 hours there are 8,000 bacteria. Find the number of bacteria after 4 hours.
- A bacteria culture grows at a rate proportional to its size. After 2 hours there are 600 bacteria and after 8 hours the count is 75,000. Find the initial population.
- The half-life of radium-226 is 1590 years. A sample of radium has a mass of 100 mg. Find the mass after 1,000 years correct to nearest miligram.
- Bismuth-210 has a half-life of 5 days. A sample originally has a mass of 800 mg. Find the mass remaining after 30 days.
- After 3 days a sample of radon-222 decayed to 58% of its original mass. What is the half-life of radon-222?
- Scientists can determine the age of ancient objects by a method called radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon-14, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates carbon-14 through food chains. When a plant or animal dies, it stops replacing its carbon and the amount of carbon-14 begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially. A parchment fragment was discovered that had about 74% as much carbon-14 radioactivity as does a plant material on Earth today. Estimate the age of the parchment.

3. Graphing solutions. Sketch the slope field for the equation

- $y' = y + t$
- $y' = t^2 - y^2$
- $y' = t^2 + y^2$
- $y' = 2$
- $y' = y$

4. Logistic equation.

- Suppose that a population grows according to a logistic model with carrying capacity 6,000 and $k = 0.0015$ per year. Write the logistic differential equation for these data. If the initial population is 1,000, find the population after 50 years.
- The Pacific halibut fishery has been modeled by the differential equation

$$y' = ky\left(1 - \frac{y}{K}\right)$$

where $y(t)$ is the biomass in kilograms at time t , the carrying capacity is estimated to be $K = 8 \times 10^7$ kg, and $k = 0.71$ per year. If the initial biomass is 2×10^7 kg, how long will it take for the biomass to reach 4×10^7 kg?

- One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the population who have heard the rumor and the fraction who have not heard the rumor. Write a differential equation that is satisfied by y . Solve the equation. A small town has 1,000 inhabitants. At 8 AM, 80 people have heard that a child was abducted by an unidentified driver of a 1965 Lincoln Continental. By noon half the town already knew it. At what time will 90% of the population have heard about children being abducted in black cars?