Practice test 2

1. **Product rule.** Differentiate the following functions:
   - $x^2 e^x$,
   - $\sqrt{x} e^x$,
   - $(2x^3 + 3)(x^4 - 2x)$,
   - $(u^{-2} + u^{-3})(u^5 - 2u^2)$,
   - $(t + e^t)(3 - \sqrt{t})$,
   - $e^x \cos x$,
   - $\cos^2(x)$,
   - $\cos^4(x)$,
   - $e^{3t} \sin(3t)$,
   - $\sin(3t) \ln t$.
2. **Product rule cont.**
   - 12.2.6
   - 12.2.10
   - A telepfone company wants to estimate the number of new residential phone lines that it will need to install during the upcoming month. At the beginning of January the company had 100,000 subscribers, each of whom had 1.2 phone lines, on average. The company estimated that its subscribership was increasing at the rate of 1000 monthly. By polling its existing subscribers, the company found that each intended to install an average of 0.01 new phone lines by the end of January. Estimate the number of new lines the company will have to install in January.
   - In 1999 the population of Richmond-Petersburg metropolitan area, VA, was 961,400, and the population was increasing at roughly 9200 people per year. The average annual income was $30,593 per capita, and this average was increasing at about $1400 per year (a little above the national average of about $1225 yearly). Estimate the rate at which total personal income was rising in the Richmond-Petersburg area in 1999.
   - A manufacturer produced bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$. Assuming $f(20) = 10,000$ and $f'(20) = -350$, find $R'(20)$.
3. **Higher derivatives and second derivative test.** Find local minimas, local maximas, and points of inflection of
   - $2x^3 - 3x^2 - 12x$
   - $2 + 3x - x^3$
   - $x^4 - 6x^2$
   - $200 + 8x^3 + x^4$
   - $3x^5 - 5x^3 + 3$
   - $x\sqrt{x+3}$
   - $3x^{2/3} - x$
   - $x^{1/3}(x + 4)$
   - $2\cos(x) - \cos(2x)$
   - $x + \cos(x)$
4. **Linearization.** Use linearization to estimate the given number:
   - $(2.001)^5$
   - $\sqrt{99.8}$
   - $(8.06)^{2/3}$
   - $1/1002$
   - $\ln 1.07$

5. **Differential equations.**

- 13.2.6 – 13.2.10
- A wanna-be climber drops a carabiner off a cliff, which hits the ground with a speed of $120 ft/s$. What is the height of the cliff?
- A car is travelling at $50 mi/h$ when the brakes are fully applied, producing a constant deceleration of $22 ft/s^2$. What is the distance covered before the car comes to a stop?
- What constant acceleration is required to increase the speed of a car from $30 mi/h$ to $50 mi/h$ in $5s$?
- A car braked with a constant deceleration of $16 ft/s^2$, producing skid marks measuring $200 ft$ before coming to stop. How fast was the car travelling when the brakes were first applied?
- A car is travelling at $100 km/h$ when the driver sees an accident $80 m$ ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup? What if the driver was talking on a cell phone and, as a result, hit the brakes $2s$ later?