Name: **Perm number:**

Final exam

Time: 3 hours

1. A farmer wants to make a rectangular field with a total area of 1800 m^2. It is surrounded by a fence. It is divided into 3 equal areas by fences. What is the shortest total length of fence this can be done with.

2. Evaluate the following integrals

   (a) $\int_0^4 (1 + 3y - y^2)dy$

   (b) $\int_1^8 \sqrt[3]{x}dx$

   (c) $\int_\pi^{2\pi} \cos\theta d\theta$

3. Find f if $f''(x) = 1 - 6x + 48x^2$, and $f(0) = 1$, and $f'(0) = 2$.

4. A city is in the shape of a rectangle. In 1995 the width of the city was 7 miles and the length of the city was 2 miles. The width of the city is growing at a rate of 1 mile in 3 years. The length of the city is growing at a rate of 1 mile in 3 years. Use the product rule to find how quickly the area of the city is growing in 1995.

5. Find the local maximum and minimum values, and the intervals of concavity of the function $h(x) = 3x^5 - 5x^3 + 3$.

6. Sketch the slope field for the equation $y' = t^2 - y^2$. Sketch the solution curve for which $y(1) = -0.5$. What is the long term behavior of this solution?

7. Solve the equation $y' = 3(100 - y)$ with the initial condition $y(0) = 90$. Find $y(1)$.

8. The Pacific halibut fishery has been modeled by the differential equation

$$y' = ky(1 - \frac{y}{K})$$

where $y(t)$ is the biomass in kilograms at time t, the carrying capacity is estimated to be $K = 8 \times 10^7$ kg, and $k = 0.71$. If the initial biomass is 2×10^7 kg, how long will it take for the biomass to reach 4×10^7 kg?

9. What is the equation of the tangent plane to $z = x^2 - y^2$ at $(x, y) = (2, 1)$?

10. Find the x and y values at the minimum of $f(x) = 2x^2 + xy + 2y^2 + 7y + 1$