Name: **Perm number:**

Practice test 3

Time: 180 minutes

1. Solve the following quadratic equations:
   - $x^2 + 5x + 6 = 0$
   - $x^2 + 6x + 9 = 0$
   - $x^2 + 2x + 6 = 0$
   - $x^2 - 4x - 7 = 0$
   - $x^2 - x - 1 = 0$
   - $x^2 + x - 1 = 0$
   - $-x^2 + 8x - 7 = 0$
   - $25x^2 + 4 = 0$
   - $3x^2 - 3x - 6 = 0$
   - $6x^2 - 7x + 2 = 0$
2. Solve the following systems of equations:
   - $\begin{cases} x + y = 6 \\ -x + y = 0 \end{cases}$
   - $\begin{cases} w + z = 4 \\ 2w + z = 5 \end{cases}$
   - $\begin{cases} x + y = 11 \\ x - y = 5 \end{cases}$
   - $\begin{cases} x + 2z = 6 \\ -2x - z = 3 \end{cases}$
   - $\begin{cases} 2z + 5w = 15 \\ 3z - w = -3 \end{cases}$
   - $\begin{cases} 2u + v = 0 \\ 3u + 2v = 2 \end{cases}$
   - $\begin{cases} 2s + 3t = 10 \\ 3s - 4t = 15 \end{cases}$
   - $\begin{cases} x + 2y = 5 \\ 2x = 4 \end{cases}$
   - $\begin{cases} 5u + 3w = 1 \\ u - 2w = 8 \end{cases}$
   - $\begin{cases} 3s + t = 2 \\ s - t = 0 \end{cases}$
3. Solve the following systems of equations:
   - $\begin{cases} x + 2y + z = 7 \\ 2x + y + 3z = 13 \\ 3x + 3y + 2z = 16 \end{cases}$
   - $\begin{cases} x + y + 2z = 1 \\ 2x - y - z = 4 \\ 3x + 2y + 5z = 3 \end{cases}$

- $\begin{cases} 2u + v - 3w = 1 \\ 5u + 2v - 6w = 5 \\ 3u - v - 4w = 7 \end{cases}$
- $\begin{cases} u + w = 0 \\ 2u + v = 1 \\ v + 2w = 1 \end{cases}$
- $\begin{cases} x + y + 3z = 0 \\ 3x + y + z = 2 \\ x - y - 4z = 1 \end{cases}$
- $\begin{cases} p + 2q + r = 3 \\ 2p + 5q - r = -4 \\ 3p - 2q - r = 5 \end{cases}$
- $\begin{cases} -2x + 2y + z = 10 \\ 2x + 3y + 2z = 1 \\ 3x + 5y + 4z = 4 \end{cases}$
- $\begin{cases} a + 2b - 4c = -4 \\ 5a + 11b - 21c = -22 \\ 3a - 2b + 3c = 11 \end{cases}$
- $\begin{cases} D + E + F = 3 \\ 2E + 3F = 4 \\ 4D + 8E - 3F = 35 \end{cases}$
- $\begin{cases} 2x + 3y - 4z = 4 \\ 3x + 4y - 3z = 8 \\ 7x + 12y - 12z = 19 \end{cases}$

4. **Basic properties of functions**

- Sketch the graph of $f(t) = 1 + t^3$ on $[1, 8]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = 1/(x + 1)$ on $[0, 2]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = 1 + \sqrt{x}$ on $[2, 4]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(t) = 1 - t^2$ on $[4, 16]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = 1 + x^{1/2}$ on $[9, 25]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(y) = 1 + y^2$ on $[1, 4]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(t) = t^2 - 3$ on $[0, 1]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = \sqrt{x - 1}$ on $[1, 2]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = x^{3/2}$ on $[1, 9]$. Find a formula for the inverse function and the domain on which it is valid.
- Sketch the graph of $f(x) = 1 - x^{5/2}$ on $[4, 16]$. Find a formula for the inverse function and the domain on which it is valid.

5. **Pythagorean theorem**

- To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the nearest meter, how many meters would be saved if it were possible to walk through the pond?
- A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the nearest tenth of a foot, between first base and third base?

- A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the nearest tenth of a foot?
- In a computer catalog, a computer monitor is listed as being 19 inches. This distance is the diagonal distance across the screen. If the screen measures 10 inches in height, what is the actual width of the screen to the nearest inch?
- The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the nearest tenth of an inch?
- Ms. Green tells you that a right triangle has a hypotenuse of 13 and a leg of 5. She asks you to find the other leg of the triangle without using paper and pencil. What is your answer?
- Generalize the above problem and give examples of at least five triples on integers a, b, c such that $a^2 + b^2 = c^2$. Can you give a method of finding such integers?
- Two joggers run 8 miles north and then 5 miles west. What is the shortest distance, to the nearest tenth of a mile, they must travel to return to their starting point?
- Oscar's dog house is shaped like a tent. The slanted sides are both 5 feet long and the bottom of the house is 6 feet across. What is the height of his dog house, in feet, at its tallest point?
- Seth made a small rectangular table for his workroom. The sides of the table are 36" and 18". If the diagonal of the table measures 43", is the table square? A table which is "square" has right angles at the corners.

6. **Word problems** 3.2.1 - 3.2.54

7. **Limits**

- Find the limit of $f(x) = x^2$ at the point $x = -1$.
- Find the limit of $f(w) = 2w^2 - 5w + 3$ at the point $w = 5$.
- Find the limit of $f(t) = (8 - 2t)/(3t - 9)$ at the point $t = 4$.
- Find the limit of $f(x) = (x - 1)/(x + 1)$ at the point $x = -1$.
- Find the limit of $f(u) = (u^2 + 2u)/(u^2 - 2u)$ at the point $u = 2$.
- Find the limit of $f(y) = (y^2 + y + 1)/(y + 1)$ at the point $y = -1$.
- Find the limit of $f(x) = (5 + 2x)/(x^3)$ at the point $x = 0$.
- Find the limit of $f(t) = (t^2 - t)/(t^3 - t)$ at the point $t = 1$.
- Find the limit of $f(x) = (11 - 7x)/(8 + 10x)$ at the point $x = -.8$.
- Find the limit of $f(t) = (t^4 - 1)/(t^3 - 1)$ at the point $t = 1$.
- Find the limit of $f(x) = (4 + 2x - x^2 + x^3)/(x^2 + 2x + 1)$ at the point $x = -1$.
- Find the limit of $f(y) = (y^4 - 4y^2)/(y^4 + 8y)$ at the point $y = -2$.

8. **Exponential and logarithmic equations**

- Solve the following equation in x: $e^{2x} = e^8$.
- Solve the following equation in x: $e^{3x-1} = e^{-3}$.
- Solve the following equation in x: $e^{x^2-1} = e$.
- Solve the following equation in x: $e^{x^2+2} = e$.
- Solve the following equation in x: $e^{x+1} = 2$.
- Solve the following equation in x: $e^{x^2+1} = 1$.
- Solve the following equation in x: $e^{-x} = e^{2x+2}$.
- Solve the following equation in x: $(e^{x-3})^2 = 2$.
- Solve the following equation in x: $2e^x = e^{4x-2}$.
- Solve the following equation in x: $(e^{2x})^{1/2} = 34$.

9. **Exponential growth.**

- **Population growth**

- We are studying the size of a bacteria culture. The bacteria culture was measured after 5 hours as having size $P(5) = 90,000$; after 8 hours it has grown to $P(8) = 100,000$. Assume the growth is exponential, i.e., $P(t)$ behaves like $P(t) = Ce^{kt}$. Determine C and k.
- We are studying the size of a bacteria culture. The bacteria culture was measured after 1 hour as having size $P(1) = 1,000,000$; after 3 hours it has grown to $P(3) = 5,000,000$. Assume the growth is exponential, i.e., $P(t)$ behaves like $P(t) = Ce^{kt}$. Determine C and k.
- We are studying the size of a fish family. The fish family was measured after 100 days as having size $P(100) = 5000$; after 200 days it has grown to $P(200) = 5500$. Assume the growth is exponential, i.e., $P(t)$ behaves like $P(t) = Ce^{kt}$. Determine C and k.
- We are studying the size of a bacteria culture. The bacteria culture was measured initially ($t = 0$) as having size $P(0) = 100,000$; after 4 hours it has grown to $P(4) = 600,000$. Assume the growth is exponential, i.e., $P(t)$ behaves like $P(t) = Ce^{kt}$. Determine C and k.

- **Half-life.**
  - How long will it take 600 grams of Plutonium 239 (half life 24,400 years) to decay to 18.75 grams?
  - How many grams of iodine 131 (half life 8.07 days) would be left after 48.42 days if you start with 25 grams?
  - Approximately how many years must a sample of Americium 241 (half life 458 years) be stored before it decays to a safe level? (i.e. 3
- **Doubling time.**
  - A country has a doubling time for its people of 20 years. If it ends up with 80 million people after 60 years, how many people did it have to start with?
  - The area covered by water lilies in a pond you own doubles in size every day. If allowed to grow the plants would cover the entire pond surface in 30 days. You had decided to do nothing until the pond is 1/2 covered. On what day will the pond be 1/2 covered? How long do you have before you must act or have your pond covered with the plants?
  - One bacteria, which divides in half to double its population once per minute, was placed in a test tube with an appropriate bacterial environment. After 59 minutes the test tube was 1/2 full. When would it be expected to fill the tube? Research and development makes a great discovery that allows the population to expand into 3 more tubes. How many more minutes can the bacteria double?
  - Human newborns usually weigh about 7.5 lb.. at birth. They double that in 6 months. If its weight continued to double every six months, how much would an average 5 year old weigh?
  - E. coli, a type of bacteria, divides once every 15 minutes. Calculate how many will exist after 24 hours if their biotic potential is infinite and its environmental resistance is zero.

10. **Derivatives** Find the derivatives of the following functions:
    - $y = x^3 - x^2 + x + 1$
    - $y = u^2 - 3u$
    - $y = -2t^2 + 6t - 3$
    - $y = -3.1x^3 + 1.1/x$
    - $y = t^3 - 6t^2 + 5t + 5t^{-4}$
    - $y = 8x^3 - 5x^2 + 4x - 3/x^2$
    - $y = 1/u + 2/u^2 - 3/u^3$
    - $y = .2x^5 + .25x^4 + .5x^2 + x + .37$
    - $y = (1/3)x^3 + (1/2)x^2 + x + 1 + 1/(2x)$
    - $y = -3x^4 + 5x^3 - 12x^2 + 1/(7x^2)$
    - $y = x(2x + 1)$
    - $y = (x + 1)(x^2 - 3)$
    - $y = (2x + 5)^2$
    - $y = (x^2 + 1)(3x + 5)$
    - $y = (x^{1/3} + x^{1/4})(1/x - 1/x^2)$
11. **Tangent lines**

- Find the equation of the tangent line to the curve $y = x^2$ at $x = 3$.
- Find the equation of the tangent line to the curve $y = -x^2$ at $(-2, -4)$.
- Find the equation of the tangent line to the curve $y = x^2/2 - 1$ at $x = -1$
- Find the equation of the tangent line to the curve $y = 1 - x^2$ at $(1, 0.$
- Find the equation of the tangent line to the curve $y = x^2 - 3x + 2$ at $x = 0$
- Find the equation of the tangent line to the curve $y = x^2 - 3x + 4$ at $(1, 2)$.
- Find the equation of the tangent line to the curve $y = x^2/2 - 2x + 3$ at $x = 4$.
- Find the equation of the tangent line to the curve $y = x^3 - x^2$ at $(1, 0)$
- Find the equation of the tangent line to the curve $y = x^3 - 8x + 4$ at $x = -2$.
- Find the equation of the tangent line to the curve $y = 1/x$ at $(1, 1)$.

12. **Optimization problems**. 8.13.1 – 8.13.20