Practice test 3 – Math 363, instructor: Pawel Gladki

Time: 180 minutes

1. Let X be a nonempty set, let $P(X)$ denote the set of all subsets of X. Show that $P(X)$ is a group with the group action defined as follows:
$$A \oplus B = (A \cup B) \setminus (A \cap B),$$
for $A, B \in P(X)$.

2. Check if the set
$$\{z \in \mathbb{C} : \Re z = \Im z\}$$
is a subgroup of the group $\mathbb{C}$.

3. Determine all left and right cosets of the subgroup $SL(2, \mathbb{Z}_2)$ in the group $GL(2, \mathbb{Z}_2)$.

4. For the group $D(4)$ find the set of generators containing possibly least elements. Is this group cyclic?

5. Determine all normal subgroups of the group $U(\mathbb{Z}_{10})$.

6. Use the isomorphism theorem to show that $\mathbb{R}^2 \cong \mathbb{R}^3/H$, where $H = \{(x, y, z) \in \mathbb{R}^3 : x + 2y = x - z = 0\}$.

7. Let
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 1 & 5 & 2 & 3 & 6 & 9 & 7 & 8 \end{pmatrix}, \, , \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 5 & 4 & 6 & 9 & 1 & 2 & 3 \end{pmatrix}.$$
Is the permutation $\tau^{-1}\sigma\tau$ even?

8. Check if the set
$$\left\{ \begin{bmatrix} a & -3b \\ b & a \end{bmatrix} \in M(2, \mathbb{Q}) : a, b \in \mathbb{Q} \right\}$$
is a subring of the ring $M(2, \mathbb{Q})$.

9. Find all units and zero divisors in the ring $\mathbb{Z}[i]$.

10. Check if the function $f : M(2, \mathbb{R}) \to M(2, \mathbb{R})$, $f(X) = A^{-1}XA$, where $A \in GL(2, \mathbb{R})$, is a ring homomorphism? If so, find its kernel and image.