Practice test 1 – Math 363, instructor: Pawel Gladki

Time: 60 minutes

1. Decide whether the following set is a subgroup of $\mathbb{C}$:

$$\mathbb{Z}[\sqrt{D}] = \{a + b\sqrt{D} : a, b \in \mathbb{Z}\},$$

   where D is a negative integer whose absolute value is not a square of an integer (e.g. $D = -5$, or $D = -11$, or $D = -12$ etc.)

2. Compute the index $(\mathbb{R}^* \times \mathbb{R}^* : \mathbb{R}^*_+ \times \mathbb{R}^*_+)$.

3. Check if the group $\mathbb{Z}_2 \times \mathbb{Z}_3$ is cyclic.

4. Determine all normal subgroups of the group $D(4)$.

5. Check that the function $\phi : \mathbb{C}^* \to \mathbb{R}^*$, $\phi(z) = |z|$ is a group homomorphism. Find the kernel and the image of ϕ.