Midterm 2 – Math 363, instructor: Pawel Gladki
Friday, August 10th, 2007.

Time: 60 minutes

1. Prove that the set

$$H = \{a \in \mathbb{R}^* : a^3 \text{ is rational}\}$$

is a subgroup of $\mathbb{R}^*$ (nonzero reals with multiplication).

2. Determine all left and right cosets of the subgroup $H = \{id, O_{120}, O_{240}\}$ in the group $D(3)$.

3. Determine orders of all the elements of the group $U(\mathbb{Z}_{12})$. Is $U(\mathbb{Z}_{12})$ cyclic?

4. Show that $\{I, -I\} \lhd GL(n, \mathbb{R})$, where I denotes the identity matrix.

5. Check if the following function is an endomorphism of the group $\mathbb{C}^*$ (nonzero complex numbers with multiplication):

$$\phi(z) = 5z.$$