Midterm 1 – Math 363, instructor: Pawel Gladki
Friday, August 3rd, 2007.

Time: 60 minutes

1. Prove the following theorem:

$$\sum_{k=1}^{n}(ak+b)=\frac{n}{2}[an+(a+2b)],$$

where a, b are some fixed real numbers.

2. Solve the equation

$$(4-3i)z^2-(2+11i)z-(5+i)=0.$$

3. Solve the following matrix equation:

$$X\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & -1 & 3 \\ 4 & 3 & 2 \\ 1 & -2 & 5 \end{bmatrix}$$

4. Let R be the relation in the set $\mathbb{Q}$ defined as follows: for two rational numbers x and y, xRy if and only if $\frac{x}{y}=t^2$ for some rational number t. Show that R is an equivalence relation, and has infinitely many equivalence classes.

5. In the set $\mathbb{R}$ of real numbers we define the "addition" as follows: $x \oplus y = \sqrt[3]{x^3+y^3}$. Find a neutral element e of this addition and show that $(\mathbb{R}, \oplus, e)$ is an Abelian group.