Final examination – Math 363, instructor: Pawel Gladki

Friday, August 17th, 2007
Time: 180 minutes

1. Check if the set $\mathbb{R}$ with "addition" defined by $a \oplus b = a + b + 1$ is a group.
2. Determine all subgroups of the group $\mathbb{Z}_4$.
3. Determine the index $(GL(n,\mathbb{R}) : SL(n,\mathbb{R}))$.
4. Is the group $\mathbb{Z}_3 \times \mathbb{Z}_3$ cyclic?
5. Determine the Cayley table for the factor group $U(\mathbb{Z}_{21})/\{1, 8, 13, 20\}$.
6. Use the isomorphism theorem to show that $\mathbb{R} \cong \mathbb{R}^2/H$, where $H = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$.
7. Let
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 1 & 5 & 2 & 3 & 6 & 9 & 7 & 8 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 5 & 4 & 6 & 9 & 1 & 2 & 3 \end{pmatrix}.$$
Is the permutation $\sigma\tau$ even?
8. Check if the set
$$\left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \in M(2,\mathbb{R}) : a, b, c \in \mathbb{R} \right\}$$
is a subring of the ring $M(2,\mathbb{R})$.
9. Find all units and zero divisors in the ring $\mathbb{Z}_8$.
10. Check if the function $f : M(2,\mathbb{R}) \to \mathbb{R}$, $f(X) = \det X$ is a ring homomorphism.