

Assignment 4

due date: Wednesday, August 1st, 2007.

The following is a list of some drill questions. As your homework, please solve the two problems marked with (*) and Problems 48, 49, and 50, pages 25-26 in your textbook.

Show that the following are equivalence relations:

1. equality relation between elements of any set,
2. “has the same birthday as” on the set of all people,
3. “is similar to” on the set of all triangles,
4. “has the same image under a function” on the elements of the domain of the function,
5. “is congruent modulo n ” on the set of integers,
6. “is parallel to” on the set of lines on the plane,
7. similarity on the set of well-orderings,
8. let $a, b, c, d \in \mathbb{N}$, and (a, b) and (c, d) be ordered pairs; the relations are $(a, b) \sim (c, d)$ if $a + d = b + c$, and $(a, b) \sim (c, d)$ if $ad = bc$,
9. (*) let $(r_n), (s_n)$ be any two Cauchy sequences of rational numbers; the relation is $(r_n) \sim (s_n)$ if the sequence $(r_n - s_n)$ has limit 0.

In each case describe the equivalence classes.

Explain why the following relations are **not** equivalences:

1. the relation “ \geq ” between the real numbers,
2. (*) the relation “has a common factor greater than 1 with” on the set of integers greater than 1,
3. the relation “is a sibling of” on the set of all humans,