

University of Saskatchewan
Department of Mathematics and Statistics
Practice Test III
June 1, 2006, Math 110.3, Instructor: Pawel Gladki, Time:180 minutes

Closed book. No calculators. No formula sheets.

Print your name clearly and write your student ID number on the opscan sheet. Encode your answers carefully on the opscan sheet. Use the provided examination booklets as a scrap paper only! Solutions in the examination booklets that are not copied onto the opscan sheet will NOT be graded!

Each question has equal mark. There is only one correct answer to each question. Total number of points: 40

1. Solve the inequality $|2x - 3| \leq 0.4$.
A) $[-1.3, 1.7]$ B) $[1.3, 1.7]$ C) $[-1.7, 1.3]$ D) $[0, 1.7]$
2. Solve the inequality $2x^2 + x \leq 1$.
A) $[\frac{1}{2}, 1]$ B) $[-\frac{1}{2}, 1]$ C) $[-1, \frac{1}{2}]$ D) $[-1, -\frac{1}{2}]$
3. Find an equation of the line that passes through the point $(1, -6)$ and is parallel to the line $x + 2y = 6$.
A) $y - 6 = -\frac{1}{2}(x - 1)$ B) $y - 1 = -\frac{1}{2}(x + 6)$
C) $y + 1 = -\frac{1}{2}(x - 6)$ D) $y + 6 = -\frac{1}{2}(x - 1)$
4. Find the distance between the points $(2, 5)$ and $(4, -7)$.
A) $2\sqrt{37}$ B) $4\sqrt{37}$ C) $4\sqrt{39}$ D) $2\sqrt{39}$
5. Evaluate $\cos(x - y)$ if $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$.
A) $\frac{3+8\sqrt{2}}{15}$ B) $\frac{3-8\sqrt{2}}{15}$ C) $\frac{3+8\sqrt{3}}{15}$ D) $\frac{3-8\sqrt{3}}{15}$
6. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin x = \tan x$.
A) $0, \pi$ B) $0, 2\pi$ C) $0, \pi, 2\pi$ D) $\pi, 2\pi$
7. Find all the values of x in the interval $[0, 2\pi]$ that satisfy the inequality $\sin x \leq \frac{1}{2}$.
A) $[0, \frac{\pi}{6}]$ B) $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$ C) $[0, \frac{\pi}{3}]$ D) $[0, \frac{\pi}{3}] \cup [\frac{2\pi}{3}, 2\pi]$

8. Find the domain of the function $f(t) = \sqrt{t} + \sqrt[3]{t}$.
 A) $[0, \infty)$ B) $(0, \infty)$ C) $(-\infty, 0]$ D) $(-\infty, 0)$
9. Which of the functions below has a graph obtained from the graph of $y = \sin x$ by reflecting it about the x axis and then shifting it 2 units upward and $\frac{\pi}{4}$ units to the left?
 A) $y = 2 + \sin(x + \frac{\pi}{4})$ B) $y = 2 + \sin(x - \frac{\pi}{4})$
 C) $y = -2 - \sin(x + \frac{\pi}{4})$ D) $y = 2 - \sin(x + \frac{\pi}{4})$
10. Find the function $g \circ f$ if $f(x) = \sin x$ and $g(x) = 1 - \sqrt{x}$.
 A) $y = 1 - \sqrt{\sin x}$ B) $y = \sin(1 - \sqrt{x})$
 C) $y = 1 - \sin x$ D) $y = \sqrt{1 - \sin x}$
11. Find the domain of the function $f \circ g$ if $f(x) = x + \frac{1}{x}$, $g(x) = \frac{x+1}{x+2}$.
 A) $(-\infty, -2) \cup (-2, \infty)$ B) $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$
 C) $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$ D) $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$
12. Find the exponential function $f(x) = Ca^x$, whose graph intersects with points $(1, 6)$ and $(3, 24)$.
 A) $y = 2 \cdot 3^x$ B) $y = 3 \cdot 3^x$
 C) $y = 3 \cdot 2^x$ D) $y = 2 \cdot 2^x$
13. Solve for x the equation $\ln x + \ln(x-1) = 1$.
 A) $\frac{1+\sqrt{1+4e}}{2}$ B) $\frac{1-\sqrt{1+4e}}{2}$ C) $\frac{1+\sqrt{1+4e}}{2}, \frac{1-\sqrt{1+4e}}{2}$ D) 0
14. Solve for x the inequality $\log_{\frac{1}{2}}(x-3) > -3$.
 A) $(-\infty, 11)$ B) $(11, \infty)$ C) $(-\infty, 3) \cup (11, \infty)$ D) $(3, 11)$
15. Find a formula for the inverse of the function $f(x) = e^{x^3}$.
 A) $y = \ln \sqrt[3]{x}$ B) $y = \sqrt[3]{\ln x}$
 C) $y = (\ln x)^3$ D) $y = 3^{\ln x}$
16. Find the range of the function $y = \frac{3x+2}{2x+3}$.
 A) $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$ B) $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$
 C) $(-\infty, 2) \cup (2, \infty)$ D) $(-\infty, 3) \cup (3, \infty)$
17. Simplify the expression $\sin(\tan^{-1} x)$.
 A) $\frac{1}{\sqrt{1+x^2}}$ B) $\frac{1}{\sqrt{1-x^2}}$ C) $\frac{x}{\sqrt{1-x^2}}$ D) $\frac{x}{\sqrt{1+x^2}}$
18. Evaluate the limit $\lim_{x \rightarrow 2} \frac{x^3-8}{x^2-4}$, if it exists.
 A) 3 B) 4 C) 0 D) **D.N.E.**
19. Evaluate the limit $\lim_{x \rightarrow 2} \frac{x^2-x+6}{x-2}$, if it exists.
 A) 5 B) $-\infty$ C) 0 D) **D.N.E.**
20. Evaluate the limit $\lim_{t \rightarrow 0} (\frac{1}{t\sqrt{t+1}} - \frac{1}{t})$, if it exists.
 A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) 0 D) **D.N.E.**

21. Evaluate the limit $\lim_{x \rightarrow 0} x^2 \cos 20\pi x$, if it exists.

- A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\sqrt{3}}{2}$ C) 0 D) **D.N.E.**

22. Find all values for c so that the function f defined by

$$f(x) = \begin{cases} (c-x)^2 & \text{if } x \leq 2 \\ 2 + c \sin\left(\frac{3\pi}{x}\right) & \text{if } x > 2 \end{cases}$$

is continuous on $(-\infty, \infty)$.

- A) $\frac{5+\sqrt{17}}{2}, \frac{5-\sqrt{17}}{2}$ B) 1, 2 C) 0 D) no such constants exist

23. Find the limit $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^6 - x}}{x^3 + 1}$.

- A) 3 B) -3 C) 0 D) ∞

24. Find the horizontal and vertical asymptotes of the curve $y = \frac{x}{x+4}$.

- A) $x = 4$ and $y = 1$ B) $y = 4$ and $x = 1$
C) $y = -4$ and $x = 1$ D) $x = -4$ and $y = 1$

25. Find the limit $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln 2}{h}$.

- A) 1 B) 2 C) $\frac{1}{2}$ D) $\frac{1}{\ln 2}$

26. Find $\frac{dy}{dx}$ if $y = (x^4 - 3x^2 + 5)^3$.

- A) $y = 3(x^4 - 3x^2 + 5)^2$
B) $y = 6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$
C) $y = 2x(x^4 - 3x^2 + 5)^3(2x^2 - 3)$
D) $y = 2x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$

27. Find $\frac{dy}{dx}$ if $y = e^{\sin 2x}$.

- A) $2 \cos 2x e^{\sin 2x}$ B) $-2 \cos 2x e^{\sin 2x}$
C) $\cos 2x e^{\sin 2x}$ D) $-\cos 2x e^{\sin 2x}$

28. Find $\frac{dy}{dx}$ if $xy^4 + x^2y = x + 3y$.

- A) $\frac{1+y^4-2xy}{4xy^3+x^2-3}$ B) $\frac{1-y^4+2xy}{4xy^3+x^2-3}$
C) $\frac{1-y^4-2xy}{4xy^3+x^2-3}$ D) $\frac{1+y^4+2xy}{4xy^3+x^2-3}$

29. Find $\frac{dy}{dx}$ if $y = (1 - x^{-1})^{-1}$.

- A) $(x-1)^{-1}$ B) $(x-1)^{-1}$
C) $(x-1)^{-2}$ D) $-(x-1)^{-2}$

30. Find $\frac{dy}{dx}$ if $y = \cot(3x^2 + 5)$.

- A) $-6x \csc^2(3x^2 + 5)$ B) $6x \csc^2(3x^2 + 5)$
C) $-6x \sec^2(3x^2 + 5)$ D) $6x \sec^2(3x^2 + 5)$

31. Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{x+1}(2-x)^5}{(x+3)^7}$.

- A) $\frac{(x-2)^4(3x^2-55x-52)}{\sqrt{x+1}(x+3)^8}$ B) $\frac{(x-2)^4(3x^2-55x-52)}{(x+1)(x+3)^8}$
C) $\frac{(x-2)^4(3x^2-55x-52)}{2\sqrt{x+1}(x+3)^8}$ D) $\frac{(x-2)^4(3x^2-55x-52)}{2(x+1)(x+3)^8}$

32. Find an equation of the tangent line to the curve $y = \sqrt{1 + 4 \sin x}$ at the point $(0, 1)$.
 A) $y = -2x + 1$ B) $y = 2x + 1$
 C) $y = x + 2$ D) $y = x - 2$
33. An equation of motion of the form $s = Ae^{-ct} \cos(\omega t + \delta)$ represents damped oscillation of an object. Find the velocity of the object.
 A) $v(t) = -Ae^{-ct}[c \sin(\omega t + \delta) + \omega \cos(\omega t + \delta)]$
 B) $v(t) = -Ae^{-ct}[c \cos(\omega t + \delta) - \omega \sin(\omega t + \delta)]$
 C) $v(t) = -Ae^{-ct}[c \sin(\omega t + \delta) - \omega \cos(\omega t + \delta)]$
 D) $v(t) = -Ae^{-ct}[c \cos(\omega t + \delta) + \omega \sin(\omega t + \delta)]$
34. Find $D^{103} \cos 2x$.
 A) $2^{103} \sin 2x$ B) $2^{103} \cos 2x$ C) $-2^{103} \sin 2x$ D) $-2^{103} \cos 2x$
35. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s . Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.
 A) $0.2m^2/s$ B) $0.3m^2/s$ C) $0.4m^2/s$ D) $0.5m^2/s$
36. Find the maximum amount by which a positive number can exceed its cube.
 A) $\frac{3}{2\sqrt{3}}$ B) $\frac{2}{3\sqrt{3}}$ C) $\frac{3}{2\sqrt{2}}$ D) $\frac{2}{3\sqrt{2}}$

In questions 37-40, consider the function defined by $f(x) = 2 - \frac{3}{x} - \frac{1}{x^2}$.

37. Find all the open intervals on which f is increasing.
 A) $(-\infty, -\frac{2}{3}) \cup (0, \infty)$ B) $(-\frac{2}{3}, 0)$
 C) $(-\infty, 0) \cup (\frac{2}{3}, \infty)$ D) $(0, \frac{2}{3})$
38. Find all the x coordinates where a local maximum or a local minimum value is obtained.
 A) $-\frac{2}{3}, 0$ B) $0, \frac{2}{3}$ C) 0 D) $-\frac{2}{3}$
39. Find all the intervals on which f is concave upward.
 A) $(-\infty, -\frac{2}{3})$ B) $(0, \infty)$ C) $(-\infty, -1)$ D) $(1, \infty)$
40. Find all the x coordinates of the points of inflection.
 A) $-\frac{2}{3}$ B) 0 C) -1 D) no such points

Answers: 1 B, 2 C, 3 D, 4 A, 5 A, 6 C, 7 B, 8 A, 9 D, 10 A, 11 C, 12 C, 13 A, 14 D, 15 B, 16 A, 17 D, 18 B, 19 D, 20 B, 21 C, 22 B, 23 A, 24 D, 25 C, 26 B, 27 A, 28 C, 29 D, 30 A, 31 C, 32 B, 33 D, 34 A, 35 B, 36 B, 37 A, 38 D, 39 C, 40 C