

University of Saskatchewan
Department of Mathematics and Statistics
Final Examination
June 6, 2006, Math 110.3, Instructor: Pawel Gladki, Time:180 minutes

Closed book. No calculators. No formula sheets.

Print your name clearly and write your student ID number on the opscan sheet. Encode your answers carefully on the opscan sheet. Use the provided examination booklets as a scrap paper only! Solutions in the examination booklets that are not copied onto the opscan sheet will NOT be graded!

Each question has equal mark. There is only one correct answer to each question. Total number of points: 40

1. Solve the equation $|2x - 1| - |x + 5| = 3$.
A) 3 B) $-\frac{7}{3}$ C) 9 D) $-\frac{7}{3}, 9$
2. Solve the inequality $x^3 - x^2 \leq 0$.
A) $(-\infty, 1)$ B) $(-\infty, 1]$ C) $(1, \infty)$ D) $[1, \infty)$
3. Find an equation of the line that passes through the point $(-1, -2)$ and is orthogonal to the line $2x + 5y + 8 = 0$.
A) $2x - 5y + 1 = 0$ B) $5x - 2y + 1 = 0$
C) $2x + 5y + 1 = 0$ D) $2x + 5y - 1 = 0$

4. Find the distance between the points (a, b) and (b, a) .
 A) $|a - b|\sqrt{2}$ B) $|2a|\sqrt{2}$ C) $|2b|\sqrt{2}$ D) $|a^2 - b^2|$
5. Evaluate $\sin 2y$ if $\sec y = \frac{5}{4}$ and y lies between 0 and 2π .
 A) $\frac{25}{24}$ B) $-\frac{25}{24}$ C) $\frac{24}{25}$ D) $-\frac{24}{25}$
6. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin 2x = \cos x$.
 A) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ B) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}$ C) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$ D) $\frac{\pi}{3}, \frac{2\pi}{3}$
7. Find all the values of x in the interval $[0, 2\pi]$ that satisfy the inequality $-1 < \tan x < 1$.
 A) $[0, \frac{\pi}{3}) \cup (\frac{2\pi}{3}, \frac{4\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi]$ B) $(0, \frac{\pi}{3}) \cup (\frac{2\pi}{3}, \frac{4\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi)$
 C) $[0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi]$ D) $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \frac{5\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$
8. Find the domain of the function $f(t) = \frac{x}{\sqrt[4]{x^2 - 5x}}$.
 A) $(-\infty, 0) \cup (5, \infty)$ B) $(0, 5)$
 C) $(-\infty, -5) \cup (0, \infty)$ D) $(-5, 0)$
9. Which of the functions below has a graph obtained from the graph of $y = \sin x$ by reflecting it about the x axis and then shifting it 3 units upward and $\frac{\pi}{4}$ units to the right?
 A) $y = 3 + \sin(x - \frac{\pi}{4})$ B) $y = 3 - \sin(x - \frac{\pi}{4})$
 C) $y = -3 - \sin(x + \frac{\pi}{4})$ D) $y = 3 - \sin(x + \frac{\pi}{4})$
10. Find the function $f \circ g$ if $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{1+x}{2+x}$.
 A) $y = \frac{2x^2+6x-5}{(x+2)(x+1)}$ B) $y = \frac{2x^2-6x+5}{(x+2)(x+1)}$
 C) $y = \frac{2x^2-6x-5}{(x+2)(x+1)}$ D) $y = \frac{2x^2+6x+5}{(x+2)(x+1)}$

11. Find the domain of the function $f \circ g$ if $f(x) = x^2$, $g(x) = \sqrt{x}$.
 A) $(-\infty, \infty)$ B) $(-\infty, 0]$
 C) $[0, \infty)$ D) $(-\infty, -2) \cup (0, \infty)$
12. Find the exact value of the expression $e^{2 \ln 3}$.
 A) 1 B) 6 C) 9 D) e^2
13. Solve for x the equation $\ln x - \ln(x - 1) = 1$.
 A) $\frac{e}{e-1}$ B) $\frac{e}{e+1}$ C) $\frac{e}{1-e}$ D) $-\frac{e}{e+1}$
14. Solve for x the inequality $\ln x > -1$.
 A) $(-\infty, \frac{1}{e})$ B) $(\frac{1}{e}, \infty)$ C) $(-\infty, -\frac{1}{e})$ D) $(-\frac{1}{e}, \infty)$
15. Find a formula for the inverse of the function $f(x) = \ln(x + 3)$.
 A) $y = e^{x-3}$ B) $y = e^x - 3$
 C) $y = e^{x+3}$ D) $y = e^x + 3$
16. Find the range of the function $y = \sqrt{3 - e^{2x}}$.
 A) $(0, \sqrt{3})$ B) $[0, \sqrt{3}]$ C) $(0, \sqrt{3}]$ D) $[0, \sqrt{3})$
17. Simplify the expression $\cos(\arcsin x)$.
 A) $\sqrt{1 + x^2}$ B) $\sqrt{1 - x^2}$ C) $1 + x^2$ D) $1 - x^2$
18. Evaluate the limit $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}$, if it exists.
 A) $\frac{2}{3}$ B) $\frac{3}{2}$ C) 0 D) **D.N.E.**
19. Evaluate the limit $\lim_{x \rightarrow 1^+} \frac{x^2 - 9}{x^2 + 2x - 3}$, if it exists.
 A) ∞ B) $-\infty$ C) 4 D) **D.N.E.**

20. Evaluate the limit $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 - x^2}}{x}$, if it exists.

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) 0 D) **D.N.E.**

21. Evaluate the limit $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x^2}$, if it exists.

A) $\frac{\sqrt{2}}{2}$ B) $-\frac{\sqrt{2}}{2}$ C) 0 D) **D.N.E.**

22. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases}$$

Where is f discontinuous?

A) at 0 B) at 3 C) at 0 and at 3 D) nowhere

23. Find the limit $\lim_{x \rightarrow -\infty} (x^4 + x^5)$.

A) 1 B) 0 C) $-\infty$ D) ∞

24. Find the horizontal and vertical asymptotes of the curve $y = \frac{\cos^2 x}{x^2}$.

A) $x = 0$ and $y = 0$ B) $x = 0$ and $y = 1$

C) $x = 0$ D) $y = 0$

25. Find the limit $\lim_{h \rightarrow 0} \frac{\cos(\pi + h) + 1}{h}$.

A) 1 B) -1 C) 0 D) $\cos 1$

26. Find $\frac{dy}{dx}$ if $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$.

A) $\frac{1}{2\sqrt{x}} - \frac{3}{4\sqrt[3]{x^7}}$ B) $\frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}}$

C) $\frac{1}{2\sqrt{x}} + \frac{3}{4\sqrt[3]{x^7}}$ D) $\frac{1}{2\sqrt{x}} + \frac{4}{3\sqrt[3]{x^7}}$

27. Find $\frac{dy}{dx}$ if $y = xe^{-\frac{1}{x}}$.
- A) $e^{-\frac{1}{x}}(\frac{1}{x+1} + 1)$ B) $e^{-\frac{1}{x}}(\frac{1}{x+1} - 1)$
 C) $e^{-\frac{1}{x}}(\frac{1}{x} + 1)$ D) $e^{-\frac{1}{x}}(\frac{1}{x} - 1)$
28. Find $\frac{dy}{dx}$ if $y = x^x$.
- A) $x^x(\ln x + 1)$ B) $x^x(\ln x - 1)$
 C) $x^x(\ln x + x)$ D) $x^x(\ln x - x)$
29. Find $\frac{dy}{dx}$ if $y = e^{e^x}$.
- A) e^{1+e^x} B) e^{x+e^x} C) e^{2x} D) $e^{e^{2x}}$
30. Find $\frac{dy}{dx}$ if $\sin(xy) = x^2 - y$.
- A) $\frac{x-y \cos(xy)}{x \cos(xy)+1}$ B) $\frac{2x-y \cos(xy)}{x \cos(xy)+1}$
 C) $\frac{x-y \sin(xy)}{x \cos(xy)+1}$ D) $\frac{2x-y \sin(xy)}{x \cos(xy)+1}$
31. Find $\frac{dy}{dx}$ if $y = x \tan^{-1}(4x)$.
- A) $\frac{4x}{1+4x} + \tan^{-1}(4x)$ B) $\frac{4x}{1-4x} + \tan^{-1}(4x)$
 C) $\frac{4x}{1+16x^2} + \tan^{-1}(4x)$ D) $\frac{4x}{1-16x^2} + \tan^{-1}(4x)$
32. Find an equation of the tangent line to the curve $y = 4 \sin^2 x$ at the point $(\frac{\pi}{6}, 1)$.
- A) $y = 2\sqrt{3}x + 1 - \frac{\pi\sqrt{3}}{3}$ B) $y = 2\sqrt{3}x + 1 + \frac{\pi\sqrt{3}}{3}$
 C) $y = 2\sqrt{3}x - 1 + \frac{\pi\sqrt{3}}{3}$ D) $y = 2\sqrt{3}x - 1 - \frac{\pi\sqrt{3}}{3}$
33. A particle moves according to a law of motion $s = f(t) = t^3 - 12t^2 + 36t$, $t \geq 0$. When is the particle speeding up?

- A) when $t \in (6, \infty)$
 B) when $t \in (2, 6)$
 C) when $t \in (2, 4) \cup (6, \infty)$
 D) when $t \in (4, 6)$
34. Find $D^{103} \sin 2x$.
 A) $2^{103} \sin 2x$ B) $2^{103} \cos 2x$ C) $-2^{103} \sin 2x$ D) $-2^{103} \cos 2x$
35. Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h . At what rate is the distance between the cars increasing two hours later?
 A) 45 mi/h B) 55 mi/h C) 65 mi/h D) 75 mi/h
36. Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^2+1}$ on the interval $[0, 2]$.
 A) $f(1) = \frac{1}{2}$ and $f(0) = 0$ B) $f(2) = \frac{2}{3}$ and $f(1) = 1$
 C) $f(2) = \frac{3}{4}$ and $f(0) = 2$ D) $f(1) = \frac{5}{6}$ and $f(0) = 5$

In questions 37-40, consider the function defined by $f(x) = \frac{x^2}{x^2-1}$.

37. Find all the open intervals on which f is decreasing.
 A) $(0, 1) \cup (1, \infty)$ B) $(-\infty, 0) \cup (0, 1)$
 C) $(0, 1)$ D) $(0, \infty)$
38. Find all the x coordinates where a local maximum or a local minimum value is obtained.
 A) 0 B) 1 C) 0 and 1 D) no local minimas or maximas

39. Find all the intervals on which f is concave upward.

A) $(-1, 1)$ B) $(-\infty, -1) \cup (1, \infty)$ C) $(-\infty, -1)$ D) $(1, \infty)$

40. Find all the x coordinates of the points of inflection.

A) -1 B) 1 C) -1 and 1 D) no such points