Closed book. No calculators. No formula sheets (except for Trigonometric Identities). Answer all questions. Each problem has equal mark - 5 points. Total number of points: 35.

Problem 1: Evaluate the following integrals by integrating by parts:

- (a) $\int e^x \cos x \, dx$,
- (b) $\int y^2 e^{-y} \, dy$,
- (c) $\int \sin(\ln z) dz$.

Problem 2: Evaluate the following integrals of trigonometric functions:

- (a) $\int \tan^7 x \sec^6 x \, \mathrm{dx}$,
- (b) $\int \sin^2 y \cos^4 y \, \mathrm{dy}$,
- (c) $\int \sin 5z \cos 3z \, dz$.

Problem 3: Evaluate the following integrals using the appropriate trigonometric substitution:

(a) $\int \frac{x^3}{\sqrt{4-x^2}} \, dx$, (b) $\int \sqrt{5-2y+y^2} \, dy$.

Problem 4: Evaluate the following integrals of rational functions:

(a) $\int \frac{1}{x^2-9} dx$, (b) $\int \frac{2y^2+y+2}{y(y-1)^2} dy$.

Problem 5: Evaluate the following integral using all available techniques:

$$\int \frac{\sin^4 x + 3\sin^2 x + 2\sin x + 2}{(1 + \sin^2 x)^2 (2 + \frac{1}{2}\sin\frac{x}{2}\cos\frac{x}{2})} \cos x \, \mathrm{dx}.$$

Problem 6: Use Simpson's rule with n = 4 on the interval [1, 5] to find an approximation for $\ln 5 = \int_1^5 \frac{1}{x} dx$.

Problem 7: A cylindrical tank whose height is 5 m and whose radius is 2 m is filled with water.

- (a) Find the work required to pump the water out of the outlet located at the top of the tank.
- (b)* Assume that the power of the pump is P = 2000W (the unit of power is 1 watt = 1 newton / 1 s). Compute time t required to empty the tank using the formula:

time = work/power

- (c)* The unit which is used by SaskPower to calculate your electricity bill is called kilowatthour (kWh) this is the amount of the power used by an appliance whose power is 1000 watt per one hour. Assume that the current rate for electricity is \$0.0795 for 1 kWh. How much does it cost to empty the tank?
- (*) If you solve (a) you may gain extra 2 points for solving (b) and (c).