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On Certain Types of Local-Global Principles in the Reduced Theory of Quadratic Forms

Będlewo, June 16, 2006

K - formally real field ($\mathbb{R}$)

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Th.: If K has two square classes, then

$$(a_1, \ldots, a_n) \cong (b_1, \ldots, b_m) \text{ iff. }$$
$$[n = m] \wedge [sgn(a_1, \ldots, a_n) = sgn(b_1, \ldots, b_m)]$$

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A concept of a 'reduced' isometry

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$(a_1, \ldots, a_n)$ - reduced quadratic form $a_i \in K^* / (\Sigma K^2 \setminus \{0\})$

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$\Psi(\underline{a})$ - pp formula, $\underline{a} = (a_1, \ldots, a_k)$:

$$\exists \underline{t} \bigwedge_{j=1}^{m} p_j(\underline{t}, \underline{a}) \in D(q_j(\underline{t}, \underline{a}), r_j(\underline{t}, \underline{a})),$$

$\underline{t} = (t_1, \ldots, t_n)$, $p_j(\underline{t}, \underline{a})$, $q_j(\underline{t}, \underline{a})$, $r_j(\underline{t}, \underline{a})$ - products of $\pm$ some of the t_i's and some of the a_l's

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P. Gładki, M. Marshall, *The pp conjecture for spaces of orderings of rational conics*, to appear in J. Algebra Appl.

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if and only if $ab \geq 0$ or $ac \geq 0$ for all real points on the curve S (a, b, c - polynomials, which are representatives of cosets)

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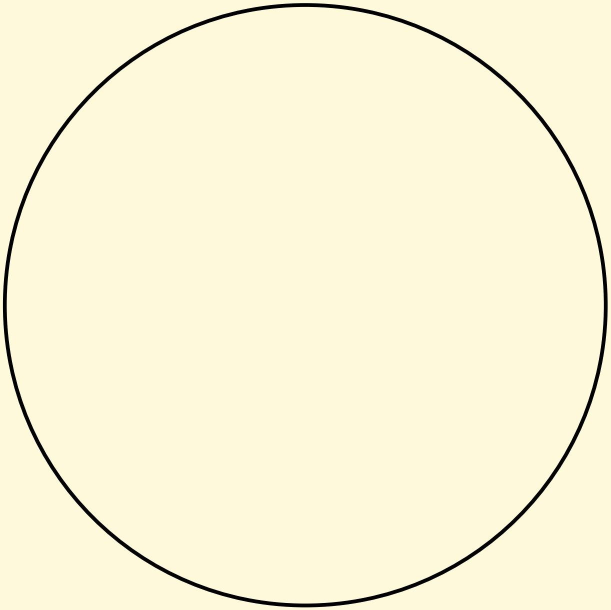
$$x^2 + y^2 = 3$$



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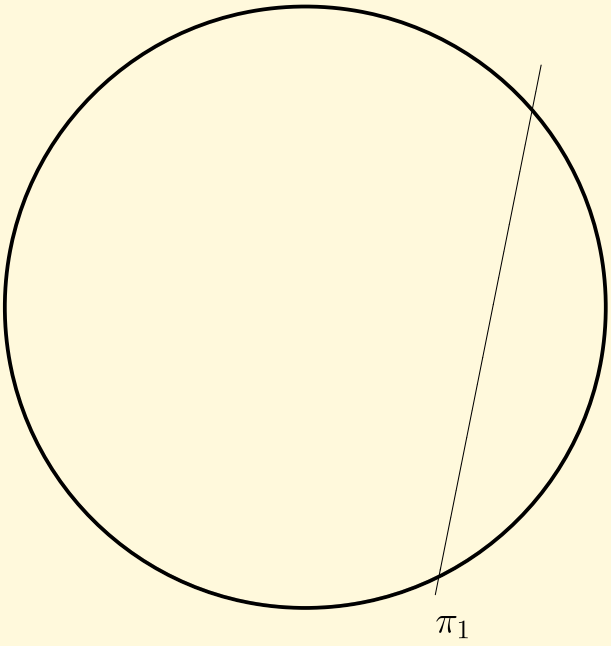

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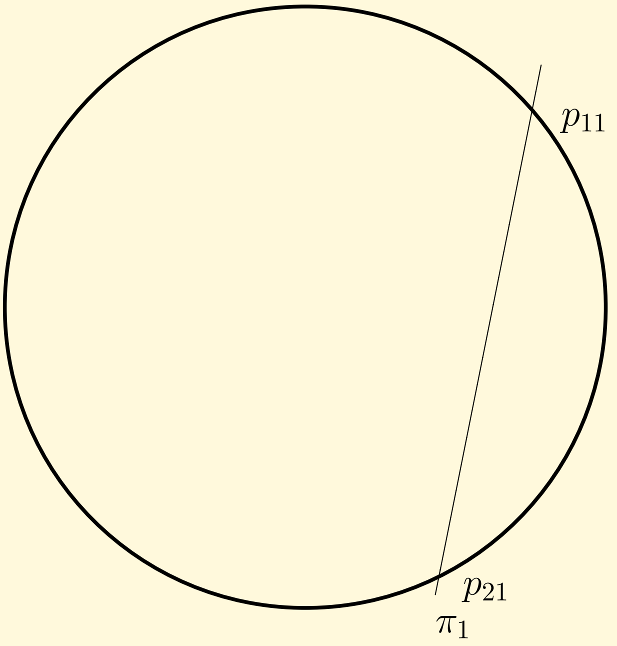

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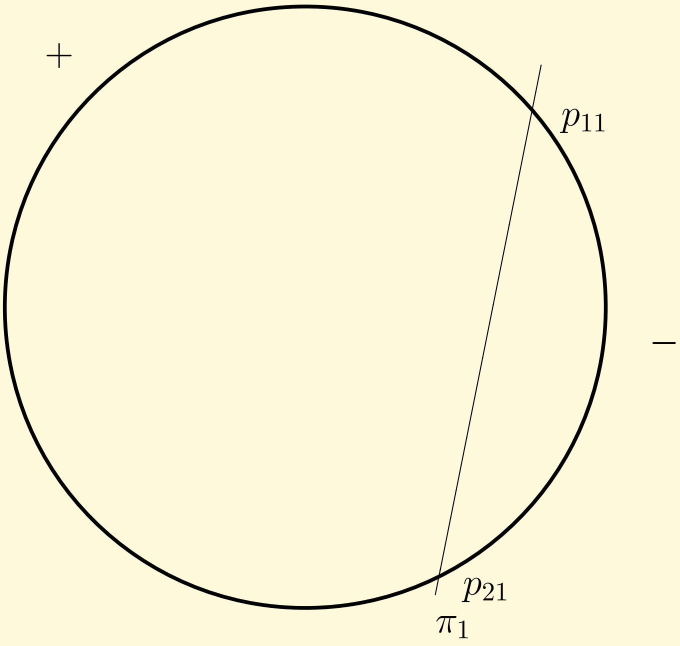

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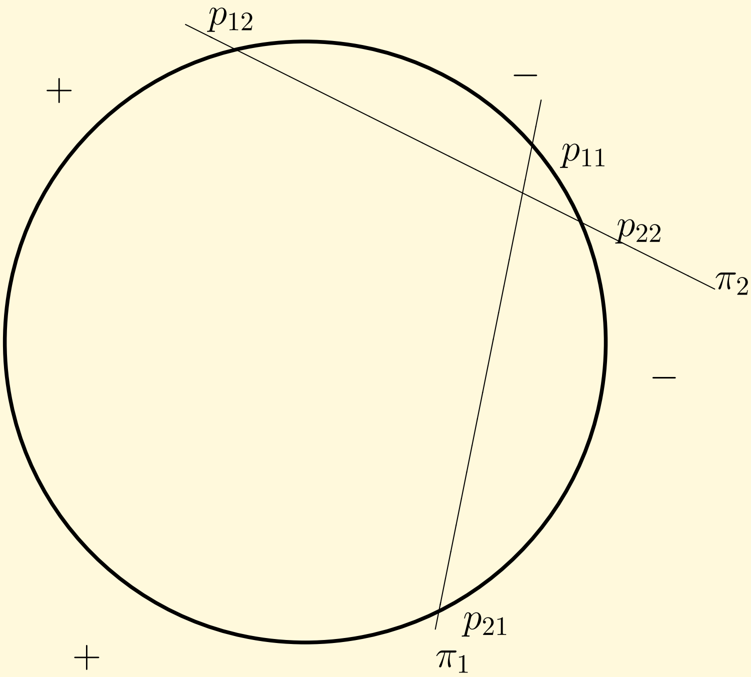

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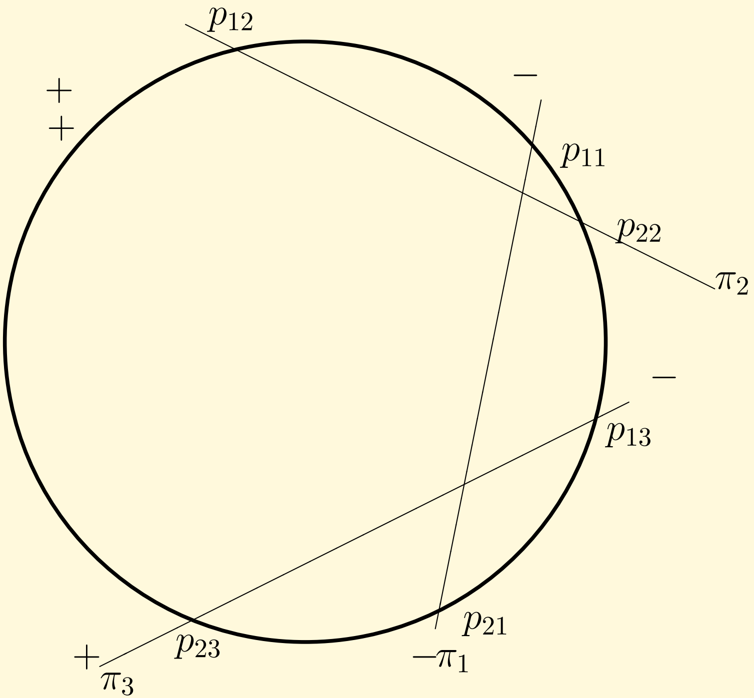

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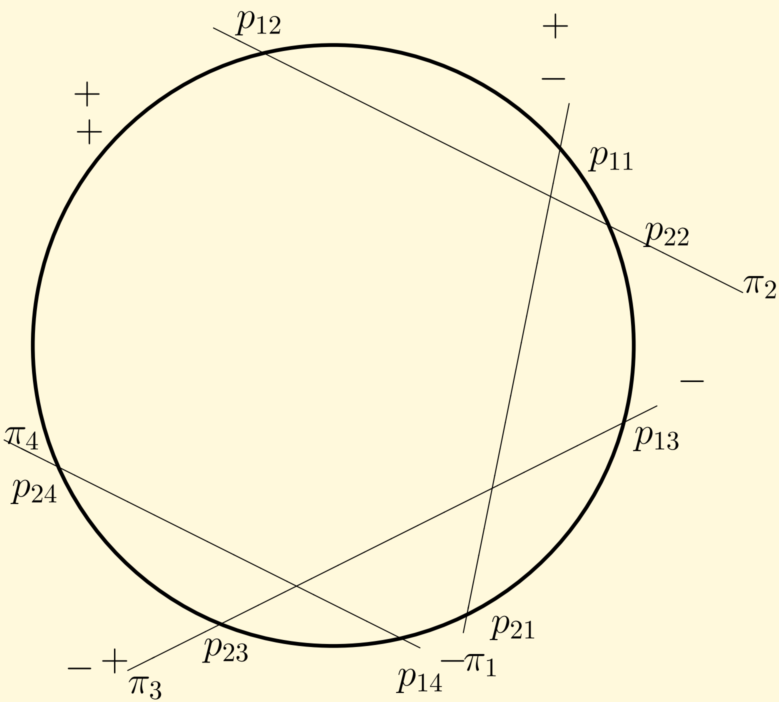

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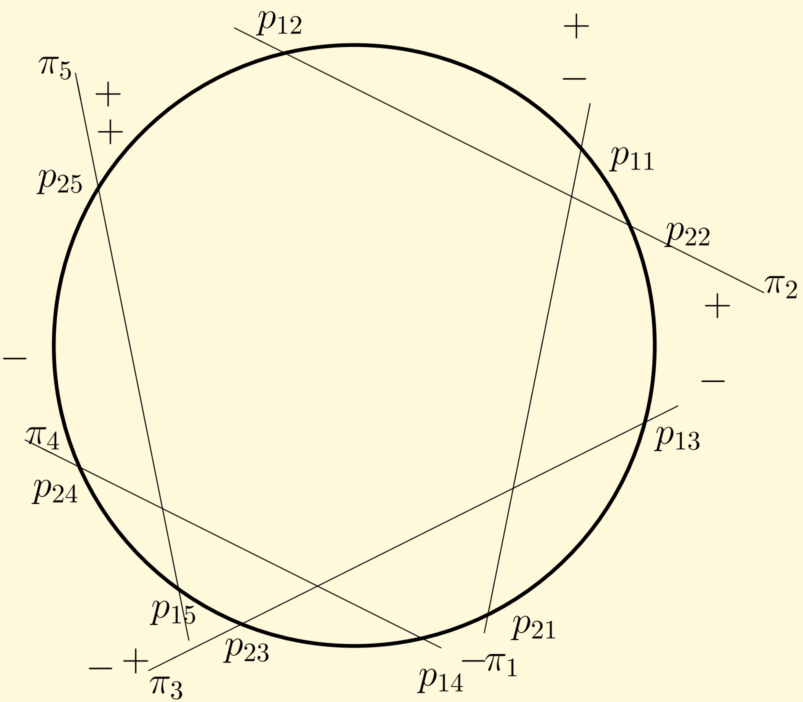

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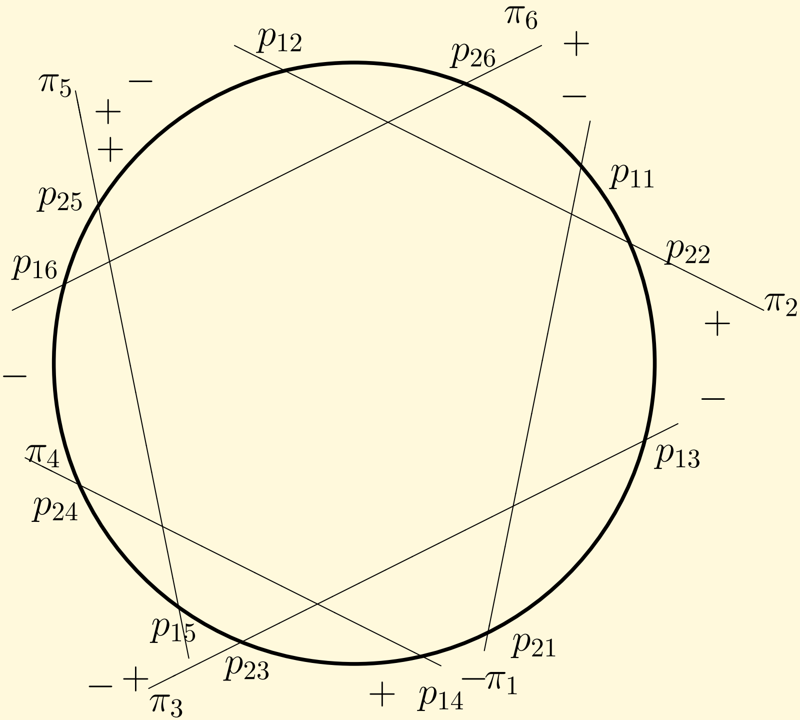
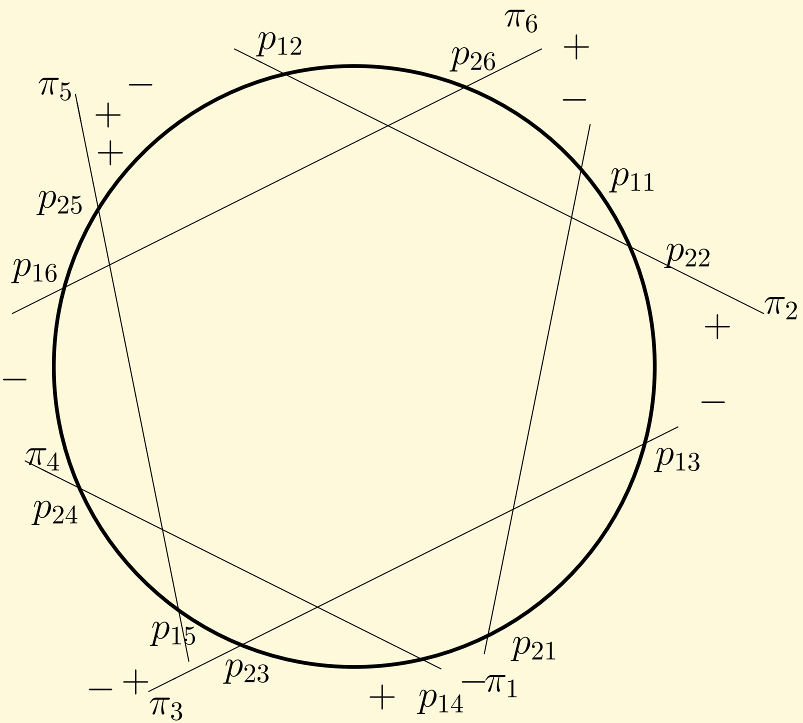


π_6
p_{12}
p_{26}
+
−
π_5
−
+
+
p_{11}
p_{25}
p_{22}
p_{16}
π_2
+
−
−
p_{13}
π_4
p_{24}
p_{15}
p_{21}
p_{23}
− +
π_3
+
p_{14}
$-\pi_1$

$$x^2 + y^2 = 3$$



$$a_1 = \pi_1 \pi_6,$$

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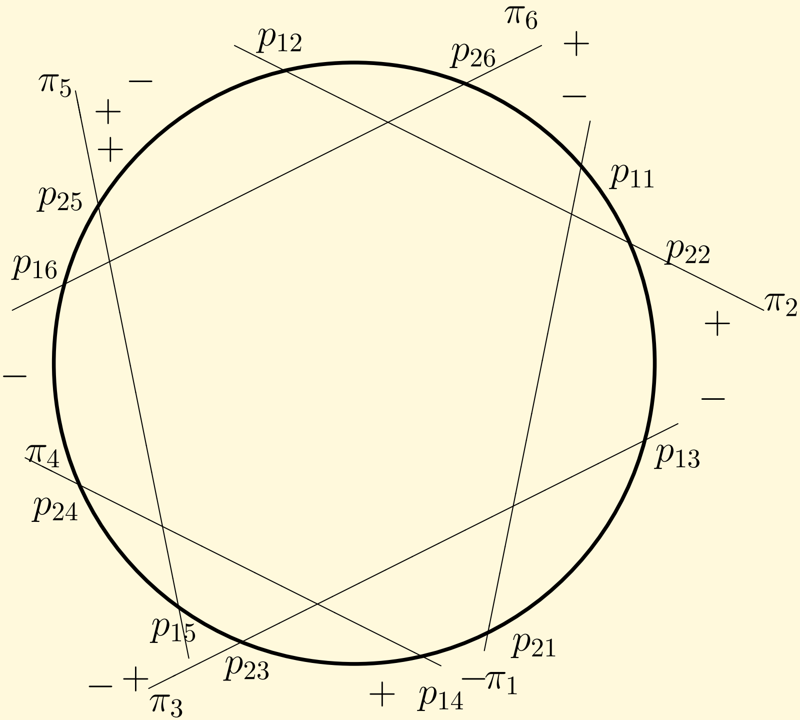
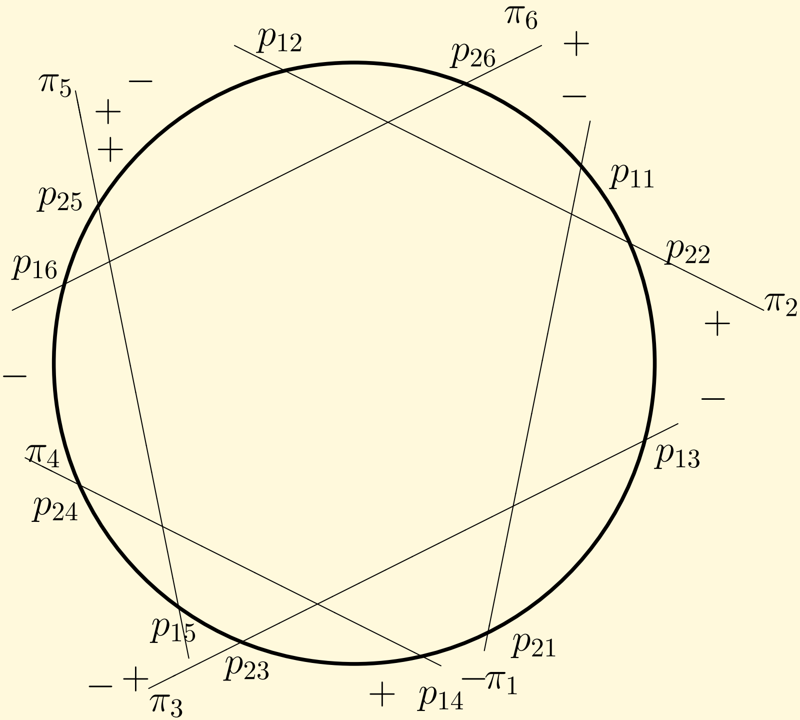


$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4,$

$$x^2 + y^2 = 3$$



$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

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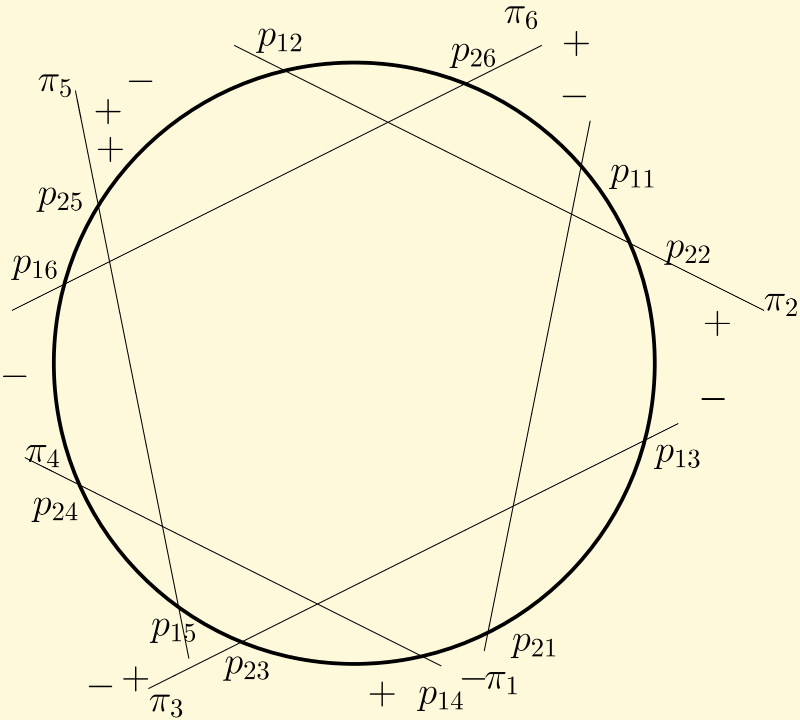

	a_1	a_2	a_3
(p_{11}, p_{22})			
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
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(p_{12}, p_{26})			
(p_{26}, p_{11})			

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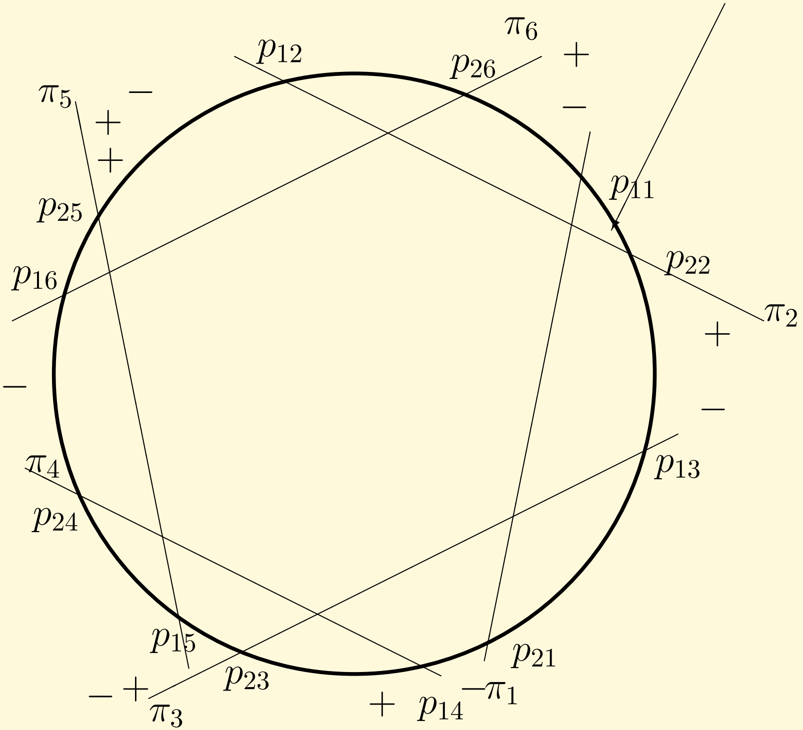

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π_1: -

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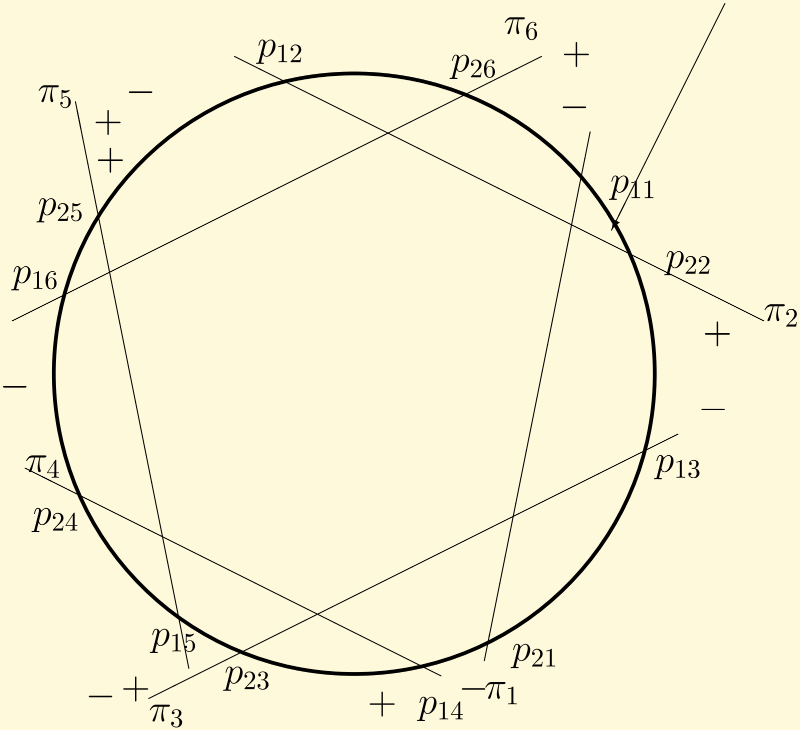

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π_2: -

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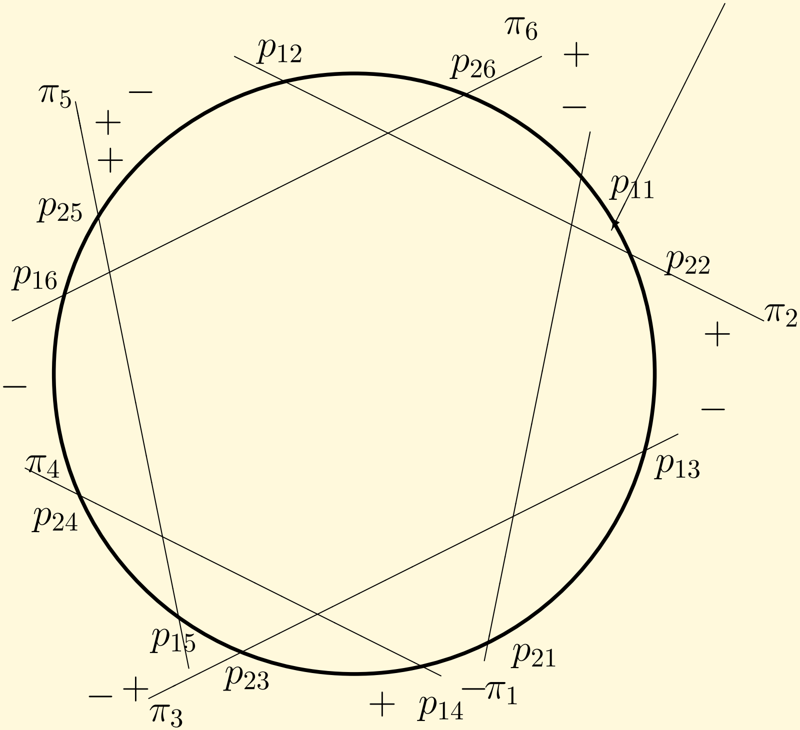

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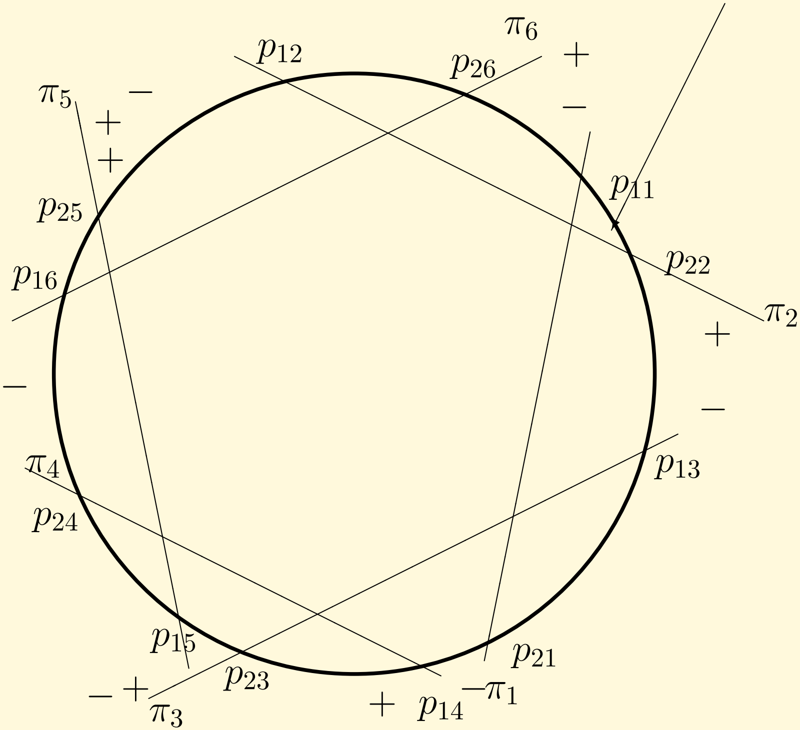

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π_2: -

π_3: $+$

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π_1: - π_4: +

π_2: -

π_3: +

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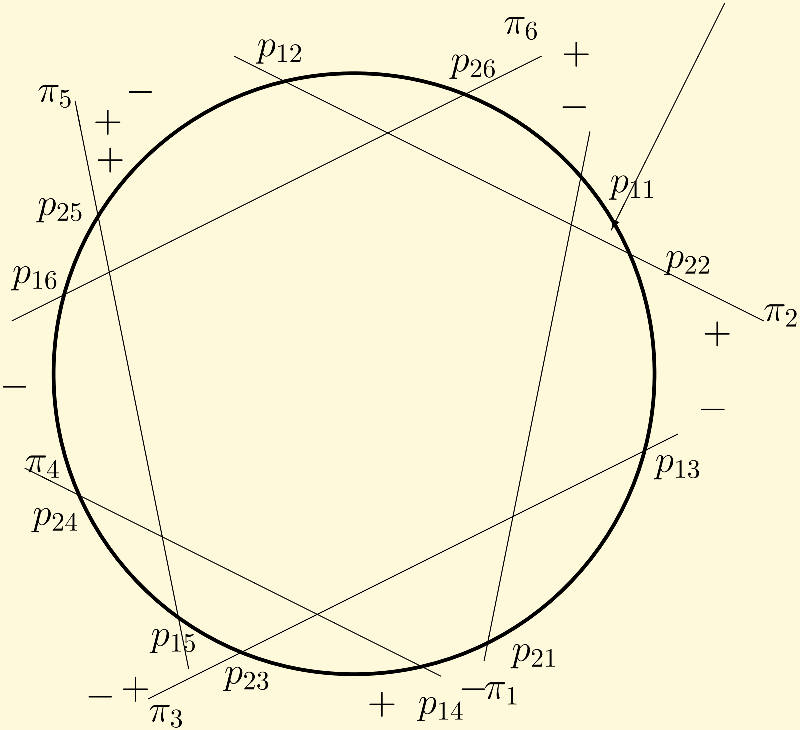

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(p_{12}, p_{26})			
(p_{26}, p_{11})			

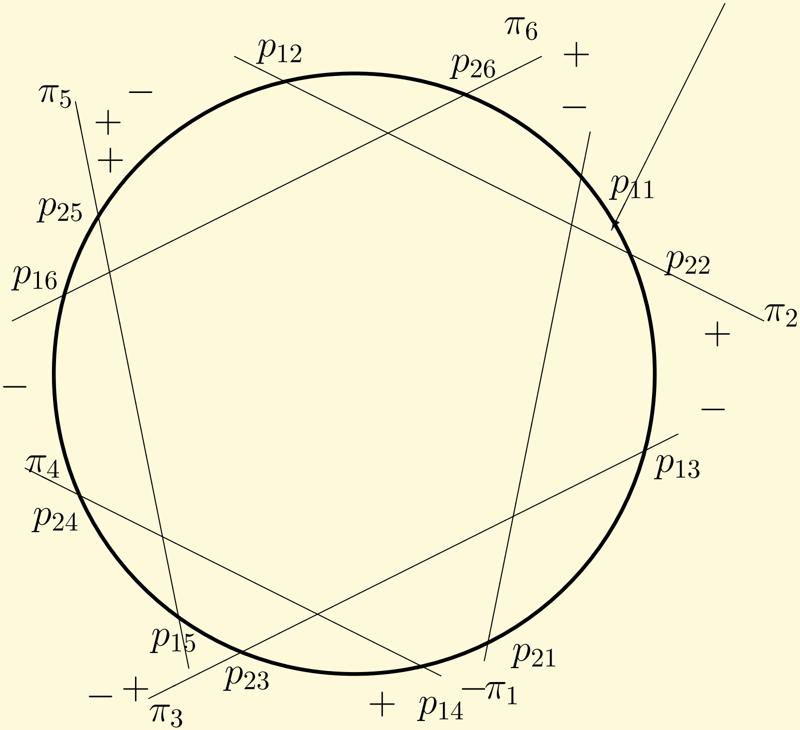
$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

π_1: - π_4: +

π_2: - π_5: +

π_3: +

$x^2 + y^2 = 3$



$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$

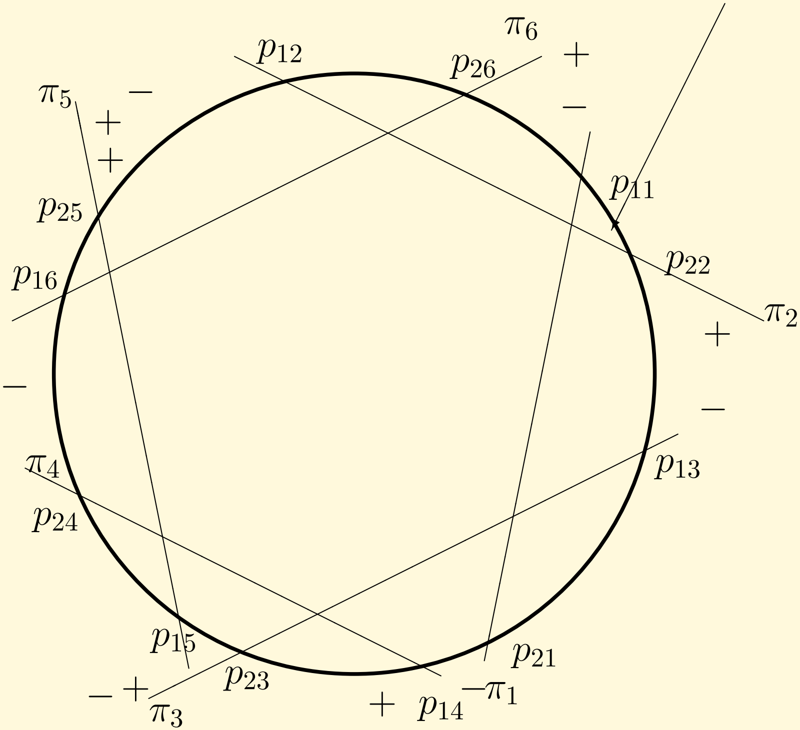
	a_1	a_2	a_3
(p_{11}, p_{22})			
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
(p_{24}, p_{16})			
(p_{16}, p_{25})			
(p_{25}, p_{12})			
(p_{12}, p_{26})			
(p_{26}, p_{11})			

π_1: - $\quad \pi_4$: $+$

π_2: - $\quad \pi_5$: $+$

π_3: $+$ $\quad \pi_6$: $+$

$$x^2 + y^2 = 3$$



$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

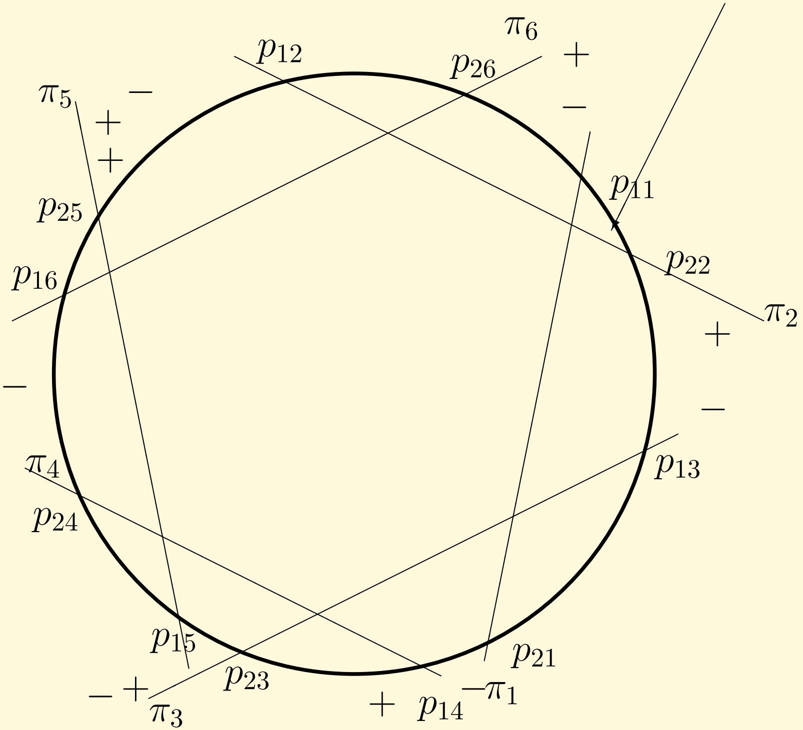
	a_1	a_2	a_3
(p_{11}, p_{22})			
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
(p_{24}, p_{16})			
(p_{16}, p_{25})			
(p_{25}, p_{12})			
(p_{12}, p_{26})			
(p_{26}, p_{11})			

π_1: - π_4: $+$

π_2: - π_5: $+$

π_3: $+$ π_6: $+$

$$x^2 + y^2 = 3$$



$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

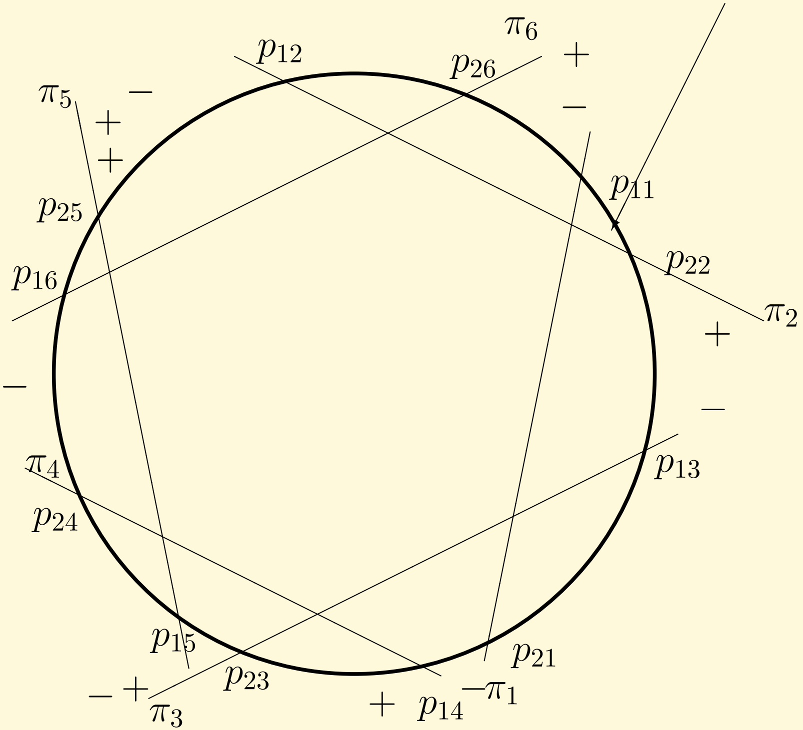
	a_1	a_2	a_3
(p_{11}, p_{22})	$-$		
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
(p_{24}, p_{16})			
(p_{16}, p_{25})			
(p_{25}, p_{12})			
(p_{12}, p_{26})			
(p_{26}, p_{11})			

π_1: - π_4: $+$

π_2: - π_5: $+$

π_3: $+$ π_6: $+$

$$x^2 + y^2 = 3$$



	a_1	a_2	a_3
(p_{11}, p_{22})	$-$	$-$	
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
(p_{24}, p_{16})			
(p_{16}, p_{25})			
(p_{25}, p_{12})			
(p_{12}, p_{26})			
(p_{26}, p_{11})			

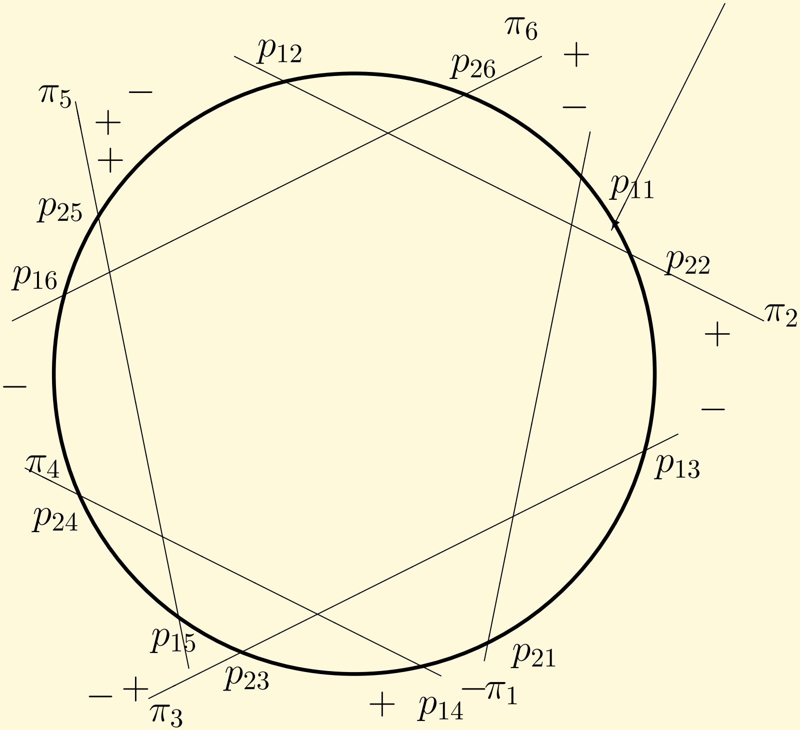
$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

π_1: - π_4: $+$

π_2: - π_5: $+$

π_3: $+$ π_6: $+$

$$x^2 + y^2 = 3$$

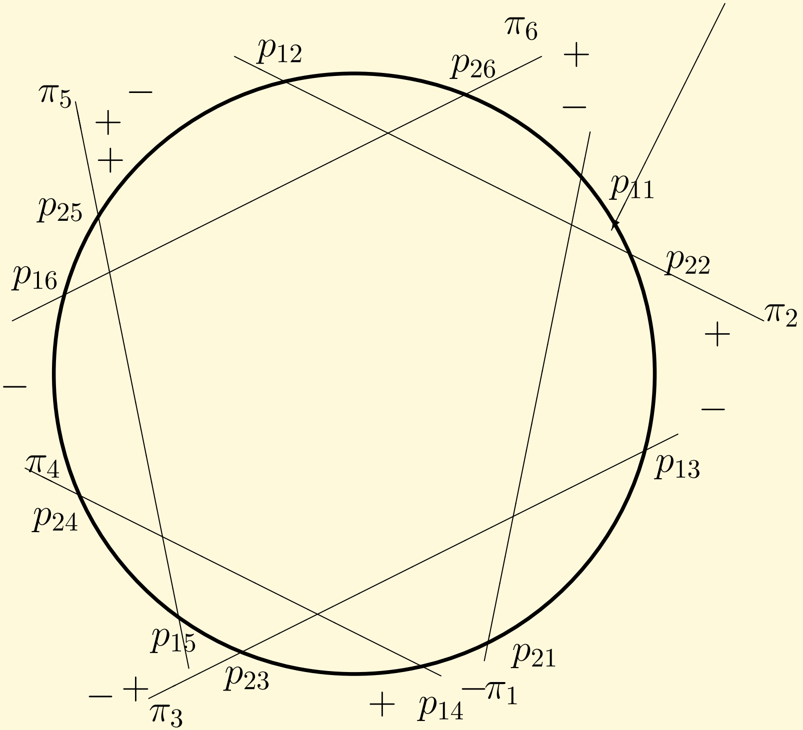


	a_1	a_2	a_3
(p_{11}, p_{22})	$-$	$-$	$-$
(p_{22}, p_{13})			
(p_{13}, p_{21})			
(p_{21}, p_{14})			
(p_{14}, p_{23})			
(p_{23}, p_{15})			
(p_{15}, p_{24})			
(p_{24}, p_{16})			
(p_{16}, p_{25})			
(p_{25}, p_{12})			
(p_{12}, p_{26})			
(p_{26}, p_{11})			

$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

π_1: - π_4: +
π_2: - π_5: +
π_3: + π_6: +

$$x^2 + y^2 = 3$$



$$a_1 = \pi_1\pi_6, \quad a_2 = \pi_1\pi_4, \quad a_3 = -\pi_1\pi_2\pi_3\pi_5$$

	a_1	a_2	a_3
(p_{11}, p_{22})	$-$	$-$	$-$
(p_{22}, p_{13})	$-$	$-$	$+$
(p_{13}, p_{21})	$-$	$-$	$-$
(p_{21}, p_{14})	$+$	$+$	$+$
(p_{14}, p_{23})	$+$	$-$	$+$
(p_{23}, p_{15})	$+$	$-$	$-$
(p_{15}, p_{24})	$+$	$-$	$+$
(p_{24}, p_{16})	$+$	$+$	$+$
(p_{16}, p_{25})	$-$	$+$	$+$
(p_{25}, p_{12})	$-$	$+$	$-$
(p_{12}, p_{26})	$-$	$+$	$+$
(p_{26}, p_{11})	$+$	$+$	$+$

	$(p_{11}; p_{22})$	$(p_{22}; p_{13})$	$(p_{13}; p_{21})$	$(p_{21}; p_{14})$	$(p_{14}; p_{23})$	$(p_{23}; p_{15})$	$(p_{15}; p_{24})$	$(p_{24}; p_{16})$	$(p_{16}; p_{25})$	$(p_{25}; p_{12})$	$(p_{12}; p_{26})$	$(p_{26}; p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

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	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

- on (p_{21}, p_{14}), (p_{24}, p_{16}), (p_{26}, p_{11}) $a_1, a_2 > 0 \Rightarrow t_1, t_2 \geq 0$

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

- on (p_{21}, p_{14}), (p_{24}, p_{16}), (p_{26}, p_{11}) $a_1, a_2 > 0 \Rightarrow t_1, t_2 \geq 0$
- near p_{23} $a_1 > 0 \Rightarrow t_1 > 0 \Rightarrow$ near p_{13} t_1 d.ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

- on (p_{21}, p_{14}), (p_{24}, p_{16}), (p_{26}, p_{11}) $a_1, a_2 > 0 \Rightarrow t_1, t_2 \geq 0$
- near p_{23} $a_1 > 0 \Rightarrow t_1 > 0 \Rightarrow$ near p_{13} t_1 d.ch.s.
- near p_{13} $a_1 a_2 > 0 \Rightarrow a_3 t_1 t_2 \geq 0$

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

- on (p_{21}, p_{14}), (p_{24}, p_{16}), (p_{26}, p_{11}) $a_1, a_2 > 0 \Rightarrow t_1, t_2 \geq 0$
- near p_{23} $a_1 > 0 \Rightarrow t_1 > 0 \Rightarrow$ near p_{13} t_1 d.ch.s.
- near p_{13} $a_1 a_2 > 0 \Rightarrow a_3 t_1 t_2 \geq 0$
- near p_{13} a_3 ch.s. $\Rightarrow t_2$ ch.s. $\Rightarrow$ near p_{23} t_2 ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	−	−	−	+	+	+	+	+	−	−	−	+
a_2	−	−	−	+	−	−	−	+	+	+	+	+
a_3	−	+	−	+	+	−	+	+	+	−	+	+

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)
- on (p_{22}, p_{13}) m_2 simultaneous ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	−	−	−	+	+	+	+	+	−	−	−	+
a_2	−	−	−	+	−	−	−	+	+	+	+	+
a_3	−	+	−	+	+	−	+	+	+	−	+	+

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)
- on (p_{22}, p_{13}) m_2 simultaneous ch.s.
- on (p_{13}, p_{21}) m_3 simultaneous ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)
- on (p_{22}, p_{13}) m_2 simultaneous ch.s.
- on (p_{13}, p_{21}) m_3 simultaneous ch.s.
- on (p_{11}, p_{21}) $m_1 + m_2 + m_3 + 1$ simultaneous ch.s.

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)
- on (p_{22}, p_{13}) m_2 simultaneous ch.s.
- on (p_{13}, p_{21}) m_3 simultaneous ch.s.
- on (p_{11}, p_{21}) $m_1 + m_2 + m_3 + 1$ simultaneous ch.s.
- at p_{11} and p_{21} s. of t_1, t_2 are the same $\Rightarrow m_1 + m_2 + m_3$ is odd

	$(p_{11};p_{22})$	$(p_{22};p_{13})$	$(p_{13};p_{21})$	$(p_{21};p_{14})$	$(p_{14};p_{23})$	$(p_{23};p_{15})$	$(p_{15};p_{24})$	$(p_{24};p_{16})$	$(p_{16};p_{25})$	$(p_{25};p_{12})$	$(p_{12};p_{26})$	$(p_{26};p_{11})$
a_1	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$
a_2	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
a_3	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

- at p_{23} and p_{13} t_2 ch.s. and t_1 d.ch.s.
- at p_{11} and p_{21} either t_1 ch.s. or t_2 ch.s. but not both
- on (p_{11}, p_{22}) t_1 ch.s. $\Leftrightarrow$ t_2 ch.s. (m_1 simultaneous ch.s.)
- on (p_{22}, p_{13}) m_2 simultaneous ch.s.
- on (p_{13}, p_{21}) m_3 simultaneous ch.s.
- on (p_{11}, p_{21}) $m_1 + m_2 + m_3 + 1$ simultaneous ch.s.
- at p_{11} and p_{21} s. of t_1, t_2 are the same $\Rightarrow$ $m_1 + m_2 + m_3$ is odd
- those are the only simultaneous sign changes

- $t_1 = u_1 q_1 \ldots q_k r_1 \ldots r_l$, $t_2 = u_2 q_1 \ldots q_k r'_1 \ldots r'_m$

- $t_1 = u_1 q_1 \dots q_k r_1 \dots r_l$, $t_2 = u_2 q_1 \dots q_k r'_1 \dots r'_m$
- simultaneous ch.s. $\rightarrow$ only at zeros of $q_1, \dots, q_k$

- $t_1 = u_1 q_1 \ldots q_k r_1 \ldots r_l$, $t_2 = u_2 q_1 \ldots q_k r'_1 \ldots r'_m$
- simultaneous ch.s. $\rightarrow$ only at zeros of $q_1, \ldots, q_k$
- for each q_i there is even number of zeros

- $t_1 = u_1 q_1 \ldots q_k r_1 \ldots r_l$, $t_2 = u_2 q_1 \ldots q_k r'_1 \ldots r'_m$
- simultaneous ch.s. $\longrightarrow$ only at zeros of $q_1, \ldots, q_k$
- for each q_i there is even number of zeros
- $m_1 + m_2 + m_3$ is even

Th.: The formula

$$\exists t_1 \exists t_2 (t_1 \in D(1, a_1) \wedge t_2 \in D(1, a_2) \wedge a_3 t_1 t_2 \in D(1, a_1 a_2))$$

holds true for every finite subset $Y \subset X_K$ iff. it hold true for every subset of orderings associated with valuations induced by $\pi_1, \ldots, \pi_6$ via the Baer-Krull correspondence

Problem 1: Give examples of elliptic curves whose coordinate rings are PID.

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Problem 2: Give an 'elementary' proof of the fact that irreducible rational polynomials intersect with rational conics without rational point transversally.

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