Math 3CI Linear Algebra Part II.

1. (a) Is x^3 in the span of $\{x^3+x, x^2+1, x+1, 2x^2\}$? If you answer yes be sure to explicitly write is as a linear combination.
(b) Is x^3 in the span of $\{x^3+x, x^2+x+1, x+1, 2x^2\}$? If you answer yes be sure to explicitly write is as a linear combination.
(c) Which polynomials lie in the span of $\{x^3+x, x^2+x+1, x+1, 2x^2\}$?

Matrices are arrays of numbers, variables (or even, as we shall see later, functions.) We ususally put parentheses around them. Here is 2×3 matrix of numbers.

$$\begin{pmatrix} 1 & 4 & \pi \\ -2 & 3 & 0 \end{pmatrix}$$

It has two rows and three columns. (Row go horizontal and columns go up and down.) We locate entries in a matrix by specifying its row and column entry. The 2-2 entry of the above matrix is 3. In the next two probelms you will develop some of the basic properties of matrices.

2. **Matrices and Matrix Multiplication.** The system of linear equations

$$\begin{array}{ccccccc} x & + & 2y & + & 3z & = & 1 \\ x & + & y & + & z & = & 2 \\ x & + & 4y & + & 7z & = & 1 \end{array}$$

can be expressed as a matrix multiplication

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Similarly,

$$\begin{array}{ccccccccc} x & + & 2y & + & 3z & + & 4w & = & 1 \\ 2x & + & 4y & + & 6z & + & 8w & = & 2 \end{array}$$

can be expressed as a matrix multiplication

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

The idea is that a row of the first matrix, times a column of the second matrix gives the corresponding row-column matrix of the product. Here are some more examples.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ 6 & 0 \end{pmatrix} = \begin{pmatrix} 17 & 1 \\ 39 & 3 \end{pmatrix}$$

Your task is to write an explanation of how matrix multiplication seems to work. **Refer to the matrix represenations of systems of equations for motivation if you haven't seen this before.** Use your explanation to calculate the following matrix products.

$$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix} = ?\ , \quad \begin{pmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 6 & 0 \\ 1 & 0 \end{pmatrix} = ?$$

3. **Reduced Row-Echelon Matrices**
(a) Find all the solutions to

$$\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(b) Find all the solutions to

$$\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The matrix in (a) and (b) is a *reduced row-echelon matrices.* Note that because of its form, it is pretty easy to record the solutions to the systems of equations in (a) and (b).
(c) For the system of equations below, change the system into an equivalent system of equations in reduced row-echelon form and use this to find all solutions.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(d) For the system of equations below, change the system into an equivalent system of equations in reduced row-echelon form and use this to find all solutions.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$