University of Saskatchewan
Department of Mathematics and Statictis
FINAL EXAMINATION
June 30, 2004 Math 116.3 Time: 180 minutes

Closed book. No calculators. No formula sheets (except for Trigonometric Identities).
Answer all questions. Each problem has equal mark - 5 points.
Total number of points: 60.

Problem 1: Find the area of the region bounded by the curves $4x + y^2 = 12$ and $y = x$.

Problem 2: Find the volume of the solid S, whose base is a circular disk with radius 5 and parallel cross-sections perpendicular to the base are squares.

Problem 3: A satellite that weighs 1000 kg is launched by a rocket from the ground to a height of 120 km. Initially the rocket has 3600 kg fuel which is used by the engine at a constant rate and finishes just as the rocket reaches the desired level. How much work is done?

Problem 4: Evaluate the following integrals:
(a) $\int \frac{2x}{(x-3)^2}\,\mathsf{dx}$,
(b) $\int \ln(y^2-1)\,\mathsf{dy}$,
(c) $\int \frac{z}{\sqrt{1-z^2}}\,\mathsf{dz}$.

Problem 5: Evaluate the following integrals:
(a) $\int \sin 4x \cos 3x\,\mathsf{dx}$,
(b) $\int \frac{y}{y^4+a^4}\,\mathsf{dy}$,
(c) $\int \frac{z^3+1}{z^3-z^2}\,\mathsf{dz}$.

Problem 6: Use the Trapezoidal Rule with $n = 4$ to approximate the integral $\int_1^2 e^{\frac{1}{x}}\,\mathsf{dx}$.

Problem 7: Prove that $\lim_{x\to 3}(x^2 + x - 12) = 0$.

Problem 8: Find a function g that agrees with $f = \frac{x^2-8x+15}{x-5}$ for all $x \neq 5$ and is continuous on $\mathbb{R}$.

Problem 9: Find the following limits:

(a) $\lim_{x\to 0^+} \frac{\ln x}{x}$,

(b) $\lim_{x\to\infty} x^{\frac{1}{x}}$.

Problem 10: Determine whether the following integrals are convergent or divergent and evaluate those that are convergent:

(a) $\int_0^4 \frac{1}{x^2+x-6}\,\mathsf{dx}$,

(b) $\int_1^{+\infty} \ln y\,\mathsf{dy}$.

Problem 11: Find the length of the curve $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{6}$.

Problem 12: Find the area of the surface obtained by rotating the curve $y = \cos 4x$, $0 \leq x \leq \frac{\pi}{12}$, about the x-axis.