

1. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \frac{n^5 - 9n^3 + 2n^2 - 98n + 8}{2n^5 - 8n^4 + 34n^3 - 2},$$

$$(b) \lim_{n \rightarrow \infty} \frac{n^3 - 6n^2 + 7n^4 - 3}{n^3 - 3n^2 + 7},$$

$$(c) \lim_{n \rightarrow \infty} \frac{(3n^2 - 2)^2}{(2n^3 + 1)(n - 1)}$$

$$(d) \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 4}}{3n - 2},$$

$$(e) \lim_{n \rightarrow \infty} \frac{\sqrt{1 + 2n^2} - \sqrt{1 + 4n^2}}{n}.$$

2. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} (\sqrt{n + 2} - \sqrt{n}),$$

$$(b) \lim_{n \rightarrow \infty} (n - \sqrt{n^2 - 5n}),$$

$$(c) \lim_{n \rightarrow \infty} (3n - \sqrt{9n^2 + 6n - 15}),$$

$$(d) \lim_{n \rightarrow \infty} (\sqrt[3]{n^3 + 4n^2} - \sqrt{n}).$$

3. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \sqrt[n]{2 \cdot 3^n + 4 \cdot 7^n},$$

$$(b) \lim_{n \rightarrow \infty} \sqrt[n]{10^n + 7^n + \left(\frac{1}{2}\right)^n},$$

$$(c) \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{1}{3}\right)^n + \left(\frac{4}{5}\right)^n + \left(\frac{3}{4}\right)^n},$$

$$(d) \lim_{n \rightarrow \infty} \frac{n}{n^2 - 1} \sin(3n + 1),$$

$$(e) \lim_{n \rightarrow \infty} \frac{\cos(n^2)}{n^2},$$

$$(f) \lim_{n \rightarrow \infty} \operatorname{arctg}\left(\frac{n^2 + 1}{n}\right),$$

$$(g) \lim_{n \rightarrow \infty} \frac{4^{n-1} - 5}{2^{2n} - 7},$$

$$(h) \lim_{n \rightarrow \infty} \frac{5 \cdot 3^{2n} - 1}{4 \cdot 9^n + 7},$$

$$(i) \lim_{n \rightarrow \infty} \frac{3 \cdot 2^{2n+2} - 10}{5 \cdot 4^{n-1} + 3}.$$

4. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n,$$

$$(b) \lim_{n \rightarrow \infty} \left(\frac{n+5}{n}\right)^n,$$

$$(c) \lim_{n \rightarrow \infty} \left(\frac{3n-1}{3n+1}\right)^{n+4},$$

$$(d) \lim_{n \rightarrow \infty} \left(\frac{5n-2}{4n+1}\right)^{-2n+9},$$

$$(e) \lim_{n \rightarrow \infty} \left(\frac{n^2-1}{n^2+2}\right)^{3n^2-4},$$

$$(f) \lim_{n \rightarrow \infty} \left(\frac{n^2-6}{2n^2+3}\right)^{n^2},$$

$$(g) \lim_{n \rightarrow \infty} \left(\frac{-6n+9}{-6n-2}\right)^{-8n+34},$$

$$(h) \lim_{n \rightarrow \infty} n(\ln(n+1) - \ln n).$$

5. Oblicz granice:

$$(a) \lim_{x \rightarrow 2} \frac{x^2 + 4}{x + 2},$$

$$(b) \lim_{x \rightarrow -\frac{1}{2}} \frac{4x^2 - 1}{2x + 1},$$

$$(c) \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2},$$

$$(d) \lim_{x \rightarrow 3} \frac{27 - x^3}{x - 3},$$

$$(e) \lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{2x - 6},$$

$$(f) \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1},$$

$$(g) \lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 9x + 20},$$

$$(h) \lim_{x \rightarrow -2} \frac{3x^2 + 5x - 2}{4x^2 + 9x + 2},$$

$$(i) \lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25},$$

$$(j) \lim_{x \rightarrow 0} \frac{\sin 3x}{4x},$$

$$(k) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{4x},$$

$$(l) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\operatorname{tg} 5x},$$

$$(m) \lim_{x \rightarrow 0} \frac{x}{\operatorname{tg} 2x},$$

$$(n) \lim_{x \rightarrow 0} \frac{x \cos 2x}{\sin 3x},$$

$$(o) \lim_{x \rightarrow 3} \sqrt{\frac{x^2 - 9}{x^2 - 4x + 3}},$$

$$(p) \lim_{x \rightarrow \pi} \frac{x \operatorname{tg} 2x}{\operatorname{tg} x \cos x}.$$

6. Oblicz pochodną funkcji:

$$(a) f(x) = \frac{1}{3}x^3 - \ln x + 4x^{-3} - \frac{3}{2}\frac{1}{x^3},$$

$$(b) f(x) = \sqrt[3]{x^2},$$

$$(c) f(x) = 2 \sin x + 4 \cos x + \frac{2}{x^5} - 2 \log_2 x,$$

$$(d) f(x) = x \sin x,$$

$$(e) f(x) = x^2 \cos x,$$

$$(f) f(x) = \sin x \cos x,$$

$$(g) f(x) = \frac{x+1}{x-3},$$

$$(h) f(x) = \operatorname{tg} x,$$

$$(i) f(x) = \operatorname{ctg} x,$$

$$(j) f(x) = \frac{x \ln x}{x^3 - 4x^2 + x - 6},$$

$$(k) f(x) = \frac{e^x(x^2 - 4x + 6)}{x - 5},$$

$$(l) f(x) = \frac{\sqrt[3]{x} - \sqrt{x}}{\sqrt[4]{x}},$$

$$(m) f(x) = \sin(2x),$$

$$(n) f(x) = (x^2 - 3x + 1)^2,$$

$$(o) f(x) = (x^5 - 3x^3 + x)^8,$$

$$(p) f(x) = \sqrt{x-4},$$

$$(q) f(x) = \frac{1}{\sqrt{x-1}},$$

$$(r) f(x) = \sin^2 x,$$

$$(s) f(x) = \cos^2 x + 5 \cos x - 2,$$

$$(t) f(x) = \sqrt{\frac{2x+3}{-6x+1}},$$

$$(u) f(x) = e^{3x+4},$$

$$(v) f(x) = 2^{\sin x},$$

$$(w) f(x) = 4^{(4 \cos x - 2x^3 + \frac{1}{x})} \cdot \sin x,$$

$$(x) f(x) = \left(1 + \frac{1}{x}\right)^{23},$$

$$(y) f(x) = \frac{3^{4x} \cdot (x^4 - 5 \operatorname{tg} x)}{\sqrt{x^5 - 1}(x^2 - 3x + 5)},$$

$$(z) f(x) = \ln \frac{5x}{\sin x},$$

$$(\acute{z}) f(x) = 10 \cdot 7^{3x^2},$$

$$(aa) f(x) = \sqrt[3]{\frac{x}{2x-3}},$$

$$(bb) f(x) = \frac{2}{3}\sqrt{x^5} - \frac{4x^2}{\sin x - \cos x},$$

$$(cc) f(x) = \log_3 \frac{x+3}{3x-7},$$

$$(dd) f(x) = 3x^7 \cdot \frac{\sin x - 8}{\cos x},$$

$$(ee) f(x) = \frac{4x^2 + \cos(5x-3)}{2x^3 - 5x^2 + 1},$$

$$(ff) f(x) = \sin x \cdot \ln \frac{2x-4}{3x-6}.$$