

Zadania z analizy matematycznej. Granice funkcji.

1. Oblicz granicę:

$$(a) \lim_{x \rightarrow 2} \frac{x^2+4}{x+2},$$

$$(b) \lim_{x \rightarrow -\frac{1}{2}} \frac{4x^2-1}{2x+1},$$

$$(c) \lim_{x \rightarrow 2} \frac{x^3-8}{x-2},$$

$$(d) \lim_{x \rightarrow 3} \frac{27-x^3}{x-3},$$

$$(e) \lim_{x \rightarrow 3} \frac{x^2-4x+3}{2x-6},$$

$$(f) \lim_{x \rightarrow -1} \frac{x^2-1}{x+1},$$

$$(g) \lim_{x \rightarrow 4} \frac{x^2-2x-8}{x^2-9x+20},$$

$$(h) \lim_{x \rightarrow -2} \frac{3x^2+5x-2}{4x^2+9x+2},$$

$$(i) \lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25},$$

$$(j) \lim_{x \rightarrow 0} \frac{\sin 3x}{4x},$$

$$(k) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{4x},$$

$$(l) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\operatorname{tg} 5x},$$

$$(m) \lim_{x \rightarrow 0} \frac{x}{\operatorname{tg} 2x},$$

$$(n) \lim_{x \rightarrow 0} \frac{x \cos 2x}{\sin 3x},$$

$$(o) \lim_{x \rightarrow 3} \sqrt{\frac{x^2-9}{x^2-4x+3}},$$

$$(p) \lim_{x \rightarrow \pi} \frac{x \operatorname{tg} 2x}{\operatorname{tg} x \cos x}.$$

2. Oblicz granice prawo- oraz lewostronne:

$$(a) \lim_{x \rightarrow 2} \frac{x}{x-2},$$

$$(b) \lim_{x \rightarrow -4} \frac{-3}{x+4},$$

$$(c) \lim_{x \rightarrow -1} \frac{4}{x^2-1},$$

$$(d) \lim_{x \rightarrow 3} \frac{2x^2}{x^2-9},$$

$$(e) \lim_{x \rightarrow 3} 4^{\frac{1}{x-3}},$$

$$(f) \lim_{x \rightarrow -2} 2^{\frac{2}{x^2-4}},$$

$$(g) \lim_{x \rightarrow -3} 5^{\frac{-1}{2x^2-18}},$$

$$(h) \lim_{x \rightarrow 3} \frac{5x^2-3x+2}{3x^2-27}.$$

3. Zbadaj ciągłość funkcji:

$$(a) f(x) = \begin{cases} \frac{x^2-25}{x-5} & \text{dla } x \neq 5 \\ 3 & \text{dla } x = 5 \end{cases},$$

$$(b) f(x) = \begin{cases} \frac{\sin 2x}{\operatorname{tg} 4x} & \text{dla } x \neq 0 \\ \frac{2}{3} & \text{dla } x = 0 \end{cases},$$

$$(c) f(x) = \begin{cases} \frac{x^2-3x+2}{x^2-8x+12} & \text{dla } x \leq 2 \\ \frac{x}{\cos x} & \text{dla } x > 2 \end{cases},$$

$$(d) f(x) = \begin{cases} \frac{3}{x+3} & \text{dla } x \geq -3 \\ 3^{\frac{1}{x^2-9}} & \text{dla } x < -3 \end{cases},$$

4. Dla jakiej wartości parametru a funkcja f jest ciągła: $f(x) = \begin{cases} \frac{x^2-2x+1}{x-1} & \text{dla } x \neq 1, \\ a & \text{dla } x = 1, \end{cases}$.