

Zadania z analizy matematycznej. Ciągi liczbowe.

1. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \frac{n^5 - 9n^3 + 2n^2 - 98n + 8}{2n^5 - 8n^4 + 34n^3 - 2},$$

$$(b) \lim_{n \rightarrow \infty} \frac{n^3 - 6n^2 + 7n^4 - 3}{n^3 - 3n^2 + 7},$$

$$(c) \lim_{n \rightarrow \infty} \frac{(3n^2 - 2)^2}{(2n^3 + 1)(n - 1)}$$

$$(d) \lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 4}}{3n - 2},$$

$$(e) \lim_{n \rightarrow \infty} \frac{\sqrt{1 + 2n^2} - \sqrt{1 + 4n^2}}{n}.$$

2. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} (\sqrt{n + 2} - \sqrt{n}),$$

$$(b) \lim_{n \rightarrow \infty} (n - \sqrt{n^2 - 5n}),$$

$$(c) \lim_{n \rightarrow \infty} (3n - \sqrt{9n^2 + 6n - 15}),$$

$$(d) \lim_{n \rightarrow \infty} (\sqrt[3]{n^3 + 4n^2} - \sqrt{n}).$$

3. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \sqrt[n]{2 \cdot 3^n + 4 \cdot 7^n},$$

$$(b) \lim_{n \rightarrow \infty} \sqrt[n]{10^n + 7^n + \left(\frac{1}{2}\right)^n},$$

$$(c) \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{1}{3}\right)^n + \left(\frac{4}{5}\right)^n + \left(\frac{3}{4}\right)^n},$$

$$(d) \lim_{n \rightarrow \infty} \frac{n}{n^2 - 1} \sin(3n + 1),$$

$$(e) \lim_{n \rightarrow \infty} \frac{\cos(n^2)}{n^2},$$

$$(f) \lim_{n \rightarrow \infty} \operatorname{arctg}\left(\frac{n^2 + 1}{n}\right),$$

$$(g) \lim_{n \rightarrow \infty} \frac{4^{n-1} - 5}{2^{2n-7}},$$

$$(h) \lim_{n \rightarrow \infty} \frac{5 \cdot 3^{2n} - 1}{4 \cdot 9^n + 7},$$

$$(i) \lim_{n \rightarrow \infty} \frac{3 \cdot 2^{2n+2} - 10}{5 \cdot 4^{n-1} + 3}.$$

4. Oblicz granice:

$$(a) \lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n,$$

$$(b) \lim_{n \rightarrow \infty} \left(\frac{n+5}{n}\right)^n,$$

$$(c) \lim_{n \rightarrow \infty} \left(\frac{3n-1}{3n+1}\right)^{n+4},$$

$$(d) \lim_{n \rightarrow \infty} \left(\frac{5n-2}{4n+1}\right)^{-2n+9},$$

$$(e) \lim_{n \rightarrow \infty} \left(\frac{n^2-1}{n^2+2}\right)^{3n^2-4},$$

$$(f) \lim_{n \rightarrow \infty} \left(\frac{n^2-6}{2n^2+3}\right)^{n^2},$$

$$(g) \lim_{n \rightarrow \infty} \left(\frac{-6n+9}{-6n-2}\right)^{-8n+34},$$

$$(h) \lim_{n \rightarrow \infty} n(\ln(n+1) - \ln n).$$