Elimination of Ramification since Abhyankar and Epp

Franz-Viktor Kuhlmann

Krakow, May 2015
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Two questions

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• What constitutes the correspondence between the given point and the corresponding point on the new variety?
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The place can single out the corresponding point on the new variety, because a birationally equivalent variety has the same function field.
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If $v = v_P$, then $Lv \cup \{\infty\}$ is the image of $L$ under $P$. 
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Valued function fields
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We take a variety defined over an arbitrary field $K$. We denote by $F$ the function field of our variety. We choose a point on the variety and some place $P$ associated with it (in general, there are many). We consider $F$ together with the valuation $\nu = \nu_P$. Further, we choose a transcendence basis $T$ of $F|K$ from the generators of the coordinate ring; then $F$ is a finite extension of the rational function field $L = K(T)$. 

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This means that we choose a new model for the function field \(F|K\) and thus a new variety. But this variety is birationally equivalent to the one we started with.
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To see in which algebraic extensions of valued fields ramification occurs, and thus the Implicit Function Theorem fails, we take any valued field \((L, v)\) and extend the valuation to the algebraic closure \(L^{\text{ac}}\) of \(L\).
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We set \(p = \text{char } Lv\) if it is positive, and \(p = 1\) otherwise.
### Absolute ramification theory

<table>
<thead>
<tr>
<th>Galois group</th>
<th>field</th>
<th>value group</th>
<th>residue field</th>
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<tbody>
<tr>
<td>$1$</td>
<td>$L^{\text{sep}}$</td>
<td>$\widetilde{\nu L}$</td>
<td>$(L^\nu)^{\text{ac}}$</td>
</tr>
<tr>
<td>$G^r$</td>
<td>$L^r$</td>
<td>$\frac{1}{p^\infty} \nu L$</td>
<td>$(L^\nu)^{\text{sep}}$</td>
</tr>
<tr>
<td>$G^i$</td>
<td>$L^i$</td>
<td>$\nu L$</td>
<td>$(L^\nu)^{\text{sep}}$</td>
</tr>
<tr>
<td>$G^d$</td>
<td>$L^h$</td>
<td>$\nu L$</td>
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</tr>
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<td>$\text{Gal } L$</td>
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</tbody>
</table>

- $L^{\text{ac}}$: purely inseparable
- $L^{\text{sep}}$: separable-algebraic closure
- $L^r$: absolute ramification field
- $L^i$: absolute inertia field
- $L^h$: absolute decomposition field (henselization)
- $\text{Galois}$, defectless
- $\text{division by } p$
- $\text{division prime to } p$
- $\text{Galois}$
where

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$\text{Char}$ denotes the character group

$\text{Hom} \left( vL^r / vL^i , (L^i v)^\times \right)$. 
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\[ F \subset K(T)^i. \]
From $L^i$ to $L'$ is the area of **tame ramification**, 

Wild ramification does not necessarily mean that the value groups change. It can also mean that we have a purely inseparable extension of the residue fields. It also refers to the case where neither value group nor residue fields change, but there is a unique extension of the valuation; in this case we have nontrivial defect.

Note that $L_r = L_{ac}$ if $p = 1$, that is, if $\text{char } L_v = 0$. In this case, all ramification is tame.
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In positive characteristic, one also has to eliminate wild ramification (explicitly or implicitly). This explains why the case of positive characteristic is so much harder. Indeed, neither local uniformization nor resolution of singularities has been proved in positive characteristic for dimensions $> 3$. 
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While all of these results explicitly or implicitly are instances of elimination of ramification, it has not been achieved in general.
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**Theorem**

Assume that \((L, v)\) and \((F, v)\) are two extensions of \((L_0, v)\), both contained in a common extension \((M, v)\).

If \(v_L \subseteq v_F\) and \((L | L_0, v)\) has only tame ramification (i.e., \(L \subseteq L_r_0\)), then \(v(L, F) = v_F\).
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Epp’s Theorem


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**Theorem**

Assume that $v_{L_0} = v_K$. If $\text{char}(K) > 0$, then assume in addition that the largest perfect subfield of $L_v$ is $K_v$. Then there is a finite extension $R'$ of $R$ such that $S/R'$ is weakly unramified over $R'$, i.e., their quotient fields $L'$ and $K'$ have the same value group.
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If $\text{char} \ L = p$, then these are Artin-Schreier extensions: $L \mid L_0$ is an Artin-Schreier extension if $L = L_0(z)$ with $z^p - z \in L_0$. If $d \in L_0$, then $L = L_0(z - d)$ and $(z - d)^p - (z - d) = z^p - z - d^p + d$. That allows us to replace any summand in $z^p - z$ that is of the form $d^p$ by its $p$-th root $d$. 

This fact has been used by several authors, including Abhyankar, Epp and myself, to find normal forms for such Artin-Schreier extensions that fit our purposes.
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It is important to note that the subgroup $G^r$ of the absolute Galois group $\text{Gal} \ L$ is a $p$-group. This means that every finite separable extension of $L^r$ is a tower of Galois extensions of degree $p$. If $\text{char} \ L = p$, then these are Artin-Schreier extensions: $L|L_0$ is an Artin-Schreier extension if $L = L_0(z)$ with $z^p - z \in L_0$.

If $d \in L_0$, then $L = L_0(z - d)$ and

$$(z - d)^p - (z - d) = z^p - z - d^p + d.$$ 

That allows us to replace any summand in $z^p - z$ that is of the form $d^p$ by its $p$-th root $d$. This fact has been used by several authors, including Abhyankar, Epp and myself, to find normal forms for such Artin-Schreier extensions that fit our purposes.
Elements of the proof

Let us discuss the case of \( \text{char } L = p \),

\[
\begin{align*}
\frac{z}{p} - \frac{z}{p} &= a_n \pi^n + a_{n-1} \pi^{n-1} + \ldots
\end{align*}
\]

with \( n \in \mathbb{Z} \) and \( a_i \in L_0 \).
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So we have that

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If $a_i \in K\nu$, then we can get rid of the term $a_i \pi^i$ by putting some $y$ with $y^p - y = a_i \pi^i$ in the extension $R'$ of $R$. 

We will be left with summands $a_i \pi^i$ where $a_i \not\in K\nu$ if $i$ is negative.

By our assumption on $L\nu$, for every $a_i \in L_0 \nu \setminus K\nu$, there is a maximal $k$ such that its $p^k$-th root is still in $L_0 \nu$.

The idea of Epp is now to replace $a_i \pi^i$ by its $p^k$-th root $a_i^{1/p^k} \pi^{i/p^k}$, putting $\pi^{i/p^k}$ into $R'$. After doing this for all negative $i$, Epp states that in the above form for $z^p - z$ we have that $a_n$ has no $p$-th root in $L_0 \nu$.

From this one easily deduces that for the new extension $L' | K'$ we have that $[L' \nu : K' \nu] = p$ and $vL' = vK'$, so $S$. $R'$ is weakly unramified over $R'$. 

Franz-Viktor Kuhlmann

Elimination of Ramification
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By our assumption on \( L_v \), for every \( a_i \in L_0 v \setminus K_v \), there is a maximal \( k \) such that its \( p^k \)-th root is still in \( L_0 v \).

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The problem is that the numbers $k$ can be different for two different indices $i$. 

The result of Epp's replacement procedure is 

$$(a^{1/p} - p + a^{-1})^{\pi - 1} = (t + t^p - t)\pi - 1 = t^p\pi - 1.$$ 

This gap went undetected for almost 30 years.
The gap in Epp’s proof

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Take $L_0v = Kv(t)$ where $t$ is transcendental over $Kv$. Take $a_{-p} = t^p$ and $a_{-1} = t^p - t$. Further, let

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So one only needs to repeat Epp’s replacement procedure a finite number of times until the unwanted combinations stop to happen.
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My own gap

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Yuri Ershov noticed the gap. In 2008, in an article called *On Henselian Rationality of Extensions* in Doklady Mathematics 78, he set out to fill the gap. Unfortunately, his article is hard to read because apparently it has been garbled by the translator.

More seriously, it contains two new gaps. One of them, surprisingly, is the the same as Epp’s gap, just at a higher level: overlooking the unwanted combinations of coefficients.
A history of gaps

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A history of gaps

So the history of elimination of ramification is at the same time a history of gaps. But Ershov’s article also contains some nice ideas. Using them, I am hopeful that I have now filled my own gap and this history of gaps has come to an end (for now).


Preprints and further information

The Valuation Theory Home Page
http://math.usask.ca/fvk/Valth.html