Intermaths Diploma Exam 2021/2022 – University of Silesia in Katowice

1. Dynamical Systems on Measures – Physical and Biological Models

- **The gamma distribution**: basic properties and applications
- **Metrics and norms in the space of measures**: Fortet–Mourier metric, Kantorovich–Wasserstein metric, total variation metric. The relationships between metrics. The Kantorovich-Rubinstein maximum principle.
- **Kantorovich-Rubinstein duality theorem.** The transport of mass problem. The space of admissible transference plan. The unit cost of shipment and the total cost. The relationship between minimization of transportation costs and the Kantorovich-Rubinstein maximum principle.
- **Markov operators - properties and applications:** Markov operator and dual operator. The relationship between Markov and dual operator. Basic properties.Discrete time stochastically perturbed dynamical system. Iterated function system
- **Invariant principle**: Semidynamical system. Limiting point of a trajectory. Properties of an invariant sets. Lyapunov-LaSalle function - properties. Invariant Principle.
- **Lasota's theorem – Lower bounded technique.** Asymptotically stable of Markov operator. Lower-bound function - properties. Lasota's theorem. Markov operators defined by a stochastic kernel. The linear Tjon - Wu equation. Collision operator - probabilistic and physical interpretation.
- **Poisson driven stochastic differential equation.** Poisson process. Definition and properties of the solution of the Poisson driven stochastic differential equation. Stochastic model of the cell cycle.

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2. Applied Graph Theory

- Notions: graph (simple, multigraph, pseudograph), isomorphism of graphs, subgraph, complement of a graph. Matrices, associated with graphs. A criterium for a sequence to be the degree sequence of a graph.

- Walk, cycle, trail, circuit, path, simple cycle. Connected components. Trees. • Bipartite graphs. The König theorem. Planar bipartite graphs.

- Planar graphs, embedding of a graph into a space. The Euler formula. Properties of planar graphs. Criteria of planarity. Dual graphs.

- Colouring of geographical maps.

- Eulerian graphs. Necessary and sufficient conditions for a graph to have a) an Eulerian cycle, b) an Eulerian path. The Fleury algorithm.

- Hamiltonian graphs. Sufficient conditions for a graph to be Hamiltonian.

- The relation between the independence number and covering number for a graph. Matching. The Hall marriage theorem.

- A metric on a graph, eccentricity, diameter, radius, central vertex, center.

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3. Introduction to Wavelets

Part I - Wavelet Transforms

- Continuous wavelet transforms
- Orthogonality relation
- Inversion formula
- Heisenberg's Uncertainty Principle
- Time and frequency windows
- Time and frequency localization
- Frames
- Wavelet frames

Part II - Orthonormal Bases of Wavelets

- Orthogonal sums of spaces
- Orthogonal projections
- Riesz systems and Riesz bases
- Orthonormal systems and Riesz systems
- Multiresolution analysis
- Construction of wavelets
- Refinement equation
- Construction of wavelets with compact supports
- Elements of tensor calculus
- Construction of wavelets in R^n
- Multiresolution analysis in R^n
- Construction of a wavelet set in R^n

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4. Mathematics of Finance, Discrete Models

- Single period securities market
- Trading strategies
- Dominant trading strategy
- Linear pricing measure
- The law of one price
- Arbitrage opportunity
- Risk neutral probability measures
- Valuation of contingent claims
- Risk neutral valuation principle
- Complete and incomplete markets
- Risk and return
- Viability of securities market
- Consumption-investment plans
- Capital Asset Pricing Model

5. Decision Making Techniques and Tools

- Linear programming: simplex method, dual problem, sensitivity analysis, geometric interpretation and applications
- Integer programming: pure and mixed integer programming, branch&bound method, applications
- Transportation problem: NW corner method, Least cost method, potential method, balanced and unbalanced transportation problem
- Quadratic programming: Lagrange method, Kuhn-Tucker conditions
- Multiple Criteria Decision Making methods: ADBASE, AHP, Topsis, Even swap
- Decision making under uncertinty and risk: Maximizing the expected profit, maximizing expected usability, min-max rule, max-min rule, Hurwicz rule, Laplace rule, Savage rule, minimal expected regret
- Goal programming

6. Computational Mathematics

- Fundamental algorithms: integer and polynomial arithmetic, Karatsuba multiplication, Euclidean algorithm.
- Fast Fourier Transform and its applications. Fast polynomial multiplication.
- Square-free decomposition, partial fraction decomposition and integration of rational functions.
- Localization, counting and isolation of polynomial roots. Bounds on roots. Sturm theorem.
- Approximation of polynomial roots.
- Elimination theory: Groebner bases and their applications.